Pearson Edexcel

Mark Scheme (Results)

Summer 2025

Pearson Edexcel International Advanced Level in Pure Mathematics P4 (WMA14)
Paper 01A

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL IAL MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)

   Marks should not be subdivided.
3. Abbreviations
   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN:
   - bod – benefit of doubt
   - ft – follow through
     - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC – special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp – decimal places
   - sf – significant figures
   - ∗ – The answer is printed on the paper or ag- answer given
   - ⎣ or d… – The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.
6. If a candidate makes more than one attempt at any question:
   a) If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   b) If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \ldots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c, \quad q \neq 0$ leading to $x = \ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
1	$\int 2x\ln(3x)\,dx = x^2\ln(3x) - \int \frac{1}{x}\times x^2\,dx$	M1
	$= ... + ...x^2$	**d**M1
	$x^2\ln(3x) - \frac{x^2}{2}$	A1
	$9\ln 9 - \frac{9}{2} - \left(\ln 3 - \frac{1}{2}\right) = 17\ln 3 - 4$	**dd**M1A1
		(5 marks)

Mark positively in this question and do not penalise poor notation such as a missing "d*x*" or spurious integral signs, "+ *c*" etc. as long as the intention is clear.

M1: Attempts integration by parts the right way round achieving $...x^2\ln(3x) \pm ...\int \frac{1}{x}\times x^2\,dx$

(You do not need to be concerned how they arrive at this)
You may see an attempt at the "DI method" e.g.

	D	I
+	$\ln(3x)$	$2x$
−	$\frac{1}{x}$	x^2

but the same condition applies e.g. they must reach $...x^2\ln(3x) \pm ...\int \frac{1}{x}\times x^2\,dx$ which may be implied by e.g. $\int 2x\ln(3x)\,dx = ...x^2\ln(3x) \pm ...x^2$
(In such cases, this mark and the following mark will be scored at the same time)
Condone e.g. $...\ln 3x.x^2 \pm ...\int \frac{1}{x}\times x^2\,dx$

dM1: For $...x^2\ln(3x) \pm ...\int \frac{1}{x}\times x^2\,dx = ... \pm ...x^2$. **Depends on the previous method mark.**

A1: Fully correct integration: $x^2\ln(3x) - \frac{x^2}{2}$ or unsimplified equivalent e.g. $2\left\{\frac{1}{2}x^2\ln(3x) - \frac{x^2}{4}\right\}$ with or without a constant of integration.
Any necessary brackets must be present unless they are implied by subsequent work.
Condone e.g. $\ln(3x).x^2 - \frac{x^2}{2}$ but not e.g. $\ln(3x^3) - \frac{x^2}{2}$ unless $\ln(3x).x^2 - \frac{x^2}{2}$ is seen first.

ddM1: Substitutes in the limits 3 **and** 1 into an expression of the form $\alpha x^2\ln(3x) + \beta x^2$ and subtracts the correct way round. Condone missing brackets e.g. $9\ln 9 - \frac{9}{2} - \ln 3 - \frac{1}{2}$
Condone slips in substitution provided the intention is clear.
They may work on each term separately e.g.

$$\left[x^2\ln(3x) - \frac{x^2}{2}\right]_1^3 = \left[x^2\ln(3x)\right]_1^3 - \left[\frac{x^2}{2}\right]_1^3 = 9\ln 9 - \ln 3 - \left(\frac{9}{2} - \frac{1}{2}\right)$$

It is dependent on the two previous method marks.

A1: $17\ln 3 - 4$ Following correct work. Condone $17\ln 3 + -4$

Note: A common slip is to differentiate $\ln(3x)$ to obtain $\frac{1}{3x}$ leading to :

$$\int 2x\ln(3x)\,dx = x^2\ln(3x) - \int \frac{1}{3x}\times x^2\,dx$$

$$= x^2\ln(3x) - \frac{x^2}{6} \rightarrow \left[x^2\ln(3x) - \frac{x^2}{6}\right]_1^3 = 9\ln 9 - \frac{9}{6} - \left(\ln 3 - \frac{1}{6}\right) = 17\ln 3 - \frac{4}{3}$$

and this scores a maximum M1dM1A0ddM1A0

Note: A common incorrect approach is to integrate in the wrong way round with various incorrect attempts to integrate $\ln(3x)$ e.g.

$$\int 2x\ln(3x)\,dx = \frac{2x}{3x} - \int \frac{2}{3x}\,dx \text{ etc.}$$

This generally scores no marks.

	Alternative by substitution $u=3x$	
	e.g. $u=3x\Rightarrow\int 2x\ln(3x)\,\mathrm{d}x=\int\frac{2}{3}u\ln u\times\frac{1}{3}\mathrm{d}u$ $=\frac{1}{9}u^2\ln u-\frac{2}{9}\int\frac{u^2}{2}\times\frac{1}{u}\mathrm{d}u$	M1
	$=...+...u^2$	**d**M1
	$\frac{1}{9}u^2\ln u-\frac{2}{9}\times\frac{u^2}{4}$	A1
	$x=1,\ 3\Rightarrow u=3,\ 9$ $9\ln 9-\frac{9}{2}-\left(\ln 3-\frac{1}{2}\right)=17\ln 3-4$	**dd**M1A1

M1: Attempts integration by parts the right way round achieving $...u^2\ln(u)\pm...\int\frac{1}{u}\times u^2\,\mathrm{d}u$

Condone e.g. $=\frac{1}{9}\ln u.u^2-\frac{2}{9}\int\frac{u^2}{2}\times\frac{1}{u}\mathrm{d}u$

dM1: For $...u^2\ln(u)\pm...\int\frac{1}{u}\times u^2\,\mathrm{d}u...\pm...u^2$. **Depends on the previous method mark.**

Condone e.g. $\frac{1}{9}\ln u.u^2-\frac{2}{9}\times\frac{u^2}{4}$

A1: Fully correct integration for their substitution: $\frac{1}{9}u^2\ln u-\frac{2}{9}\times\frac{u^2}{4}$ or unsimplified equivalent e.g.

$\frac{2}{9}\left\{\frac{1}{2}u^2\ln u-\frac{u^2}{4}\right\}$ with or without a constant of integration.

Condone e.g. $\frac{1}{9}\ln u.u^2-\frac{2}{9}\times\frac{u^2}{4}$ but not e.g. $\frac{1}{9}\ln u^3-\frac{2}{9}\times\frac{u^2}{4}$ unless $\frac{1}{9}\ln u.u^2-\frac{2}{9}\times\frac{u^2}{4}$ is seen first.

Any necessary brackets must be present unless they are implied by subsequent work.

ddM1: Substitutes in the limits 9 **and** 3 into an expression of the form $\alpha u^2\ln(u)+\beta u^2$ or reverts back to x and substitutes in the limits 3 **and** 1 and subtracts the correct way round.

Condone missing brackets e.g. $9\ln 9-\frac{9}{2}-\ln 3-\frac{1}{2}$

Condone slips in substitution provided the intention is clear.

They may work on each term separately e.g.

$$\left[\frac{1}{9}u^2\ln u-\frac{u^2}{18}\right]_3^9=\left[\frac{1}{9}u^2\ln u\right]_3^9-\left[\frac{u^2}{18}\right]_3^9=9\ln 9-\ln 3-\left(\frac{9}{2}-\frac{1}{2}\right)$$

It is dependent on the two previous method marks.

A1: $17\ln 3-4$

	Alternative by substitution $u=\ln(3x)$**.**	
	e.g. $u=\ln(3x)\Rightarrow\int 2x\ln(3x)\,\mathrm{d}x=\int\frac{2}{3}\mathrm{e}^{u}u\times\frac{1}{3}\mathrm{e}^{u}\mathrm{d}u=\frac{2}{9}\int u\mathrm{e}^{2u}\mathrm{d}u$ $=\frac{1}{9}u\mathrm{e}^{2u}-\frac{1}{9}\int\mathrm{e}^{2u}\mathrm{d}u$	M1
	$=...+...\mathrm{e}^{2u}$	**d**M1
	$\frac{1}{9}u\mathrm{e}^{2u}-\frac{1}{18}\mathrm{e}^{2u}$	A1
	$x=1,\ 3\Rightarrow u=\ln 3,\ \ln 9$ $9\ln 9-\frac{9}{2}-\left(\ln 3-\frac{1}{2}\right)=17\ln 3-4$	**dd**M1A1

M1: Attempts integration by parts the right way round achieving $...u\mathrm{e}^{2u}\pm...\int\mathrm{e}^{2u}\mathrm{d}u$

dM1: For $...u\mathrm{e}^{2u}\pm...\int\mathrm{e}^{2u}\mathrm{d}u=...\pm...\mathrm{e}^{2u}$. **Depends on the previous method mark.**

A1: Fully correct integration for their substitution: $\frac{1}{9}u\mathrm{e}^{2u}-\frac{1}{18}\mathrm{e}^{2u}$ or unsimplified equivalent e.g. $\frac{2}{9}\left(\frac{1}{2}u\mathrm{e}^{2u}-\frac{1}{4}\mathrm{e}^{2u}\right)$ with or without a constant of integration.

Any necessary brackets must be present unless they are implied by subsequent work.

ddM1: Substitutes in the limits ln 9 **and** ln 3 into an expression of the form $\alpha u\mathrm{e}^{2u}+\beta\mathrm{e}^{2u}$ or reverts back to x and substitutes in the limits 3 **and** 1 and subtracts the correct way round.

Condone missing brackets e.g. $9\ln 9-\frac{9}{2}-\ln 3-\frac{1}{2}$

Condone slips in substitution provided the intention is clear.
They may work on each term separately e.g.

$$\left[\frac{1}{9}u\mathrm{e}^{2u}-\frac{1}{18}\mathrm{e}^{2u}\right]_{\ln 3}^{\ln 9}=\left[\frac{1}{9}u\mathrm{e}^{2u}\right]_{\ln 3}^{\ln 9}-\left[\frac{1}{18}\mathrm{e}^{2u}\right]_{\ln 3}^{\ln 9}=9\ln 9-\ln 3-\left(\frac{9}{2}-\frac{1}{2}\right)$$

It is dependent on the two previous method marks.

A1: $17\ln 3-4$

Question Number	Scheme	Marks
2(a)	$\frac{dV}{dx}=3x^2$	M1 (B1 on EPEN)
	$\frac{dx}{dt}=\frac{dx}{dV}\times\frac{dV}{dt}=\frac{5}{3x^2}$	**d**M1A1
		(3)
	Alternative 1	
	$V=x^3\Rightarrow x=V^{\frac{1}{3}}\Rightarrow\frac{dx}{dV}=\frac{1}{3}V^{-\frac{2}{3}}$	M1 (B1 on EPEN)
	$\frac{dx}{dt}=\frac{dx}{dV}\times\frac{dV}{dt}=\frac{5}{3V^{\frac{2}{3}}}=\frac{5}{3x^2}$	**d**M1A1
	Alternative 2	
	$\frac{dV}{dt}=5\Rightarrow V=5t+c$	M1 (B1 on EPEN)
	$V=x^3\Rightarrow x^3=5t+c\Rightarrow 3x^2\frac{dx}{dt}=5$ or $V=x^3\Rightarrow x^3=5t+c\Rightarrow 3x^2=5\frac{dt}{dx}$	**d**M1
	$\frac{dx}{dt}=\frac{5}{3x^2}$	A1
(b)	$\frac{dx}{dt}=\frac{5}{3\times4^2}=\frac{5}{48}$	M1A1
		(2)
		(5 marks)

Note this is now being marked as M1dM1A1 not B1M1A1

(a) **Ignore any units associated with any of the expressions/values in this question.**

M1: Differentiates $V=x^3$ to obtain $\frac{dV}{dx}=...x^2$ (seen or implied)

dM1: Attempts to use $\frac{dx}{dt}=\frac{dx}{dV}\times\frac{dV}{dt}$ or equivalent e.g. $\frac{dV}{dt}=\frac{dx}{dt}\times\frac{dV}{dx}$ with their $\frac{dV}{dx}$ and $\frac{dV}{dt}$ where $\frac{dV}{dx}=kx^2$ and $\frac{dV}{dt}$ is a constant to obtain an expression of the form $\frac{\alpha}{x^2}$ oe.

May be implied by their working.

Depends on the previous mark.

A1: $\frac{dx}{dt}=\frac{5}{3x^2}$ or equivalent e.g. $\frac{5}{3}x^{-2}$ or $5\times\frac{1}{3x^2}$. Apply isw once a correct expression is seen.

Alternative 1:

M1: Differentiates $x = V^{\frac{1}{3}}$ to obtain $\frac{dx}{dV} = ...V^{-\frac{2}{3}}$ (seen or implied)

dM1: Attempts to use $\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt}$ or equivalent e.g. $\frac{dV}{dt} = \frac{dx}{dt} \times \frac{dV}{dx}$ with their $\frac{dx}{dV}$ and $\frac{dV}{dt}$ where $\frac{dx}{dV} = kV^{-\frac{2}{3}}$ and $\frac{dV}{dt}$ is a constant and changes back to x to obtain an expression of the form $\frac{\alpha}{x^2}$ oe. May be implied by their working.

Depends on the previous mark.

A1: $\frac{dx}{dt} = \frac{5}{3x^2}$ or equivalent e.g. $\frac{5}{3}x^{-2}$ or $5 \times \frac{1}{3x^2}$. Apply isw once a correct expression is seen.

Alternative 2:

M1: Uses $\frac{dV}{dt} = 5$ and integrates wrt t to obtain $V = 5t + c$ or $V = 5t$

dM1: Replaces V with x^3 and differentiates wrt t or wrt x to obtain $\alpha x^2 \frac{dx}{dt} = \beta$ or $\alpha x^2 = \beta \frac{dt}{dx}$

Depends on the previous mark.

A1: $\frac{dx}{dt} = \frac{5}{3x^2}$ or equivalent e.g. $\frac{5}{3}x^{-2}$ or $5 \times \frac{1}{3x^2}$. Apply isw once a correct expression is seen.

(b)

M1: Substitutes $x = 4$ into their expression for $\frac{dx}{dt}$

A1: $\frac{5}{48}$ or the exact equivalent. Ignore any units, correct or incorrect, and just look for the value. Do not allow e.g. 0.104… unless the correct exact value is seen previously, then apply isw.

Question Number	Scheme	Marks
3	$8y\dfrac{dy}{dx}=xe^{-y^2} \rightarrow \int 8ye^{y^2}dy=\int x\ dx$	B1
	$4e^{y^2}=\dfrac{x^2}{2}\ (+c)$	M1A1
	$4=\dfrac{5^2}{2}+c \Rightarrow c=...$	**d**M1
	$y^2=\ln\left(\dfrac{x^2-17}{8}\right)$	A1
		(5 marks)

B1: Separates the variables correctly with some indication of integration.

Allow any correct separation e.g. $8\int \dfrac{y}{e^{-y^2}}dy=\int x\ dx$, $\int ye^{y^2}dy=\dfrac{1}{8}\int x\ dx$, $\int \dfrac{y}{e^{-y^2}}dy=\int \dfrac{x}{8}\ dx$

Award for any of the above with at least one integral sign. If there are no integral signs then there must be an attempt to integrate at least one side.

Condone the omission of dx or dy as long as the intention is clear.

M1: For integrating $...ye^{y^2}$ correctly to obtain $...e^{y^2}$

May be implied by a substitution e.g. $u=y^2 \Rightarrow \int 8ye^{y^2}dy=\int \dfrac{8y}{2y}e^u du=4e^u\ (+c)$

Incorrect attempts to integrate $...ye^{y^2}$ e.g. by parts will generally score no more marks.

A1: Fully correct equation with or without a constant of integration e.g.

$4e^{y^2}=\dfrac{x^2}{2}\ (+c)$, $8e^{y^2}=x^2\ (+c)$, $e^{y^2}=\dfrac{x^2}{8}\ (+c)$ etc.

dM1: Depends on the first method mark.

Substitutes in $y=0$ and $x=5$ and proceeds to find a value for c.

A1: Correct equation: $y^2=\ln\left(\dfrac{x^2-17}{8}\right)$ oe e.g.

$y^2=\ln\left(\dfrac{x^2}{8}-\dfrac{17}{8}\right)$, $y^2=\ln\dfrac{1}{8}\left(x^2-17\right)$, $y^2=\ln\left(x^2-17\right)-\ln 8$ etc.

Brackets must be present if necessary e.g. in $y^2=\ln\left(\dfrac{x^2}{8}-\dfrac{17}{8}\right)$ but not in e.g. $y^2=\ln\dfrac{x^2-17}{8}$

Apply isw once the correct answer is seen.

Question Number	Scheme	Marks
4(a)	$4^{\frac{1}{2}}\left(1+\frac{5}{4}x\right)^{\frac{1}{2}}$	B1
	$\left(1+\frac{5}{4}x\right)^{\frac{1}{2}}=1+\left(\frac{1}{2}\right)\left(\frac{5}{4}x\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(\frac{5}{4}x\right)^2+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}\left(\frac{5}{4}x\right)^3+...$	M1A1
	$4^{\frac{1}{2}}\left(1+\frac{5}{4}x\right)^{\frac{1}{2}}=2+\frac{5}{4}x-\frac{25}{64}x^2+\frac{125}{512}x^3$	A1A1
		(5)

(a)

B1: Obtains $4^{\frac{1}{2}}(1+...x)^{\frac{1}{2}}$ or e.g. $2(1+...x)^{\frac{1}{2}}$ or e.g. $\sqrt{4}(1+...x)^{\frac{1}{2}}$ which may be implied by subsequent work.

M1: Attempts the binomial expansion of $(1+kx)^{\frac{1}{2}}$ to get the **third** or **fourth** term **unsimplified** with an acceptable structure.

The correct binomial coefficient must be combined with x^2 or x^3 which may be unsimplified.

Look for $\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}...x^2$ or $\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}...x^3$ o.e. (you do not need to be concerned with their $\frac{5}{4}$ which may be 1 or e.g. if they have a negative sign in front of the whole term)

Do not allow notation such as $\begin{pmatrix}\frac{1}{2}\\2\end{pmatrix}$, $\begin{pmatrix}\frac{1}{2}\\3\end{pmatrix}$ or ${}^{\frac{1}{2}}C_2$, ${}^{\frac{1}{2}}C_3$ unless interpreted correctly in further work.

May be implied by correct coefficients with x^2 or x^3

A1: Correct third or fourth term unsimplified for $\left(1+\frac{5}{4}x\right)^{\frac{1}{2}}$ e.g. with the correct sign.

$$\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(\frac{5}{4}x\right)^2 \quad \text{or} \quad \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}\left(\frac{5}{4}x\right)^3$$

Do not condone missing brackets e.g. $\frac{5}{4}x^2$ or $\frac{5}{4}x^3$ unless they are implied by subsequent work.

A1: Two correct simplified terms of $\frac{5}{4}x, -\frac{25}{64}x^2, \frac{125}{512}x^3$ which may be listed.

Condone $\frac{5}{4}x^1$ for this mark.

Condone coefficients given as decimals e.g. $1.25x, -0.390625x^2, 0.24414...x^3$

A1: $2+\frac{5}{4}x-\frac{25}{64}x^2+\frac{125}{512}x^3$ which may be listed and isw once a correct answer is seen.

Condone coefficients given as decimals e.g. $2+1.25x-0.390625x^2+0.244140625x^3$

Do not condone $\frac{5}{4}x^1$ for this mark and do not condone $...+-\frac{25}{64}x^2+...$

Note that if their "$4^{\frac{1}{2}}$" is incorrect they can score a maximum of B0M1A1A0A0

(a) Alternative by direct expansion:

$$(4+5x)^{\frac{1}{2}} = 4^{\frac{1}{2}} + \left(\frac{1}{2}\right)4^{-\frac{1}{2}}(5x) + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)}{2}4^{-\frac{3}{2}}(5x)^2 + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}4^{-\frac{5}{2}}(5x)^3 + \ldots$$

B1: For $4^{\frac{1}{2}}+\ldots$ or $\sqrt{4}+\ldots$ or $2+\ldots$

M1: Attempts the binomial expansion of $(4+5x)^{\frac{1}{2}}$ to get the **third** or **fourth** term **unsimplified** with an acceptable structure.

The correct binomial coefficient must be combined with x^2 or x^3 and the correct power of 4 which may be unsimplified.

Look for $\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}4^{-\frac{3}{2}}\ldots x^2$ or $\frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}4^{-\frac{5}{2}}\ldots x^3$ o.e. (you do not need to be concerned with their 5 which may be 1 or e.g. if they have a negative sign in front of the whole term)

Do not allow notation such as $\begin{pmatrix}\frac{1}{2}\\2\end{pmatrix}$, $\begin{pmatrix}\frac{1}{2}\\3\end{pmatrix}$ or ${}^{\frac{1}{2}}C_2$, ${}^{\frac{1}{2}}C_3$ unless interpreted correctly in further work.

A1: Correct third or fourth term unsimplified e.g.

$\frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)}{2}4^{-\frac{3}{2}}(5x)^2$ or $\frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}4^{-\frac{5}{2}}(5x)^3$

Do not condone missing brackets e.g. $5x^2$ or $5x^3$ unless they are implied by subsequent work.

A1: Two correct simplified terms of $\frac{5}{4}x, -\frac{25}{64}x^2, \frac{125}{512}x^3$ which may be listed.

Condone $\frac{5}{4}x^1$ for this mark.

Condone coefficients given as decimals e.g. $1.25x, -0.390625x^2, 0.24414\ldots x^3$

A1: $2+\frac{5}{4}x-\frac{25}{64}x^2+\frac{125}{512}x^3$ which may be listed and isw once a correct answer is seen.

Condone coefficients given as decimals e.g. $2+1.25x-0.390625x^2+0.244140625x^3$

Do not condone $\frac{5}{4}x^1$ for this mark and do not condone $\ldots+-\frac{25}{64}x^2+\ldots$

Any attempts to use a Maclaurin series should be sent to review.

(b)	$\lvert x \rvert \leqslant \frac{4}{5}$	B1
		(1)
(c)(i)	$2-\frac{5}{4}x-\frac{25}{64}x^2-\frac{125}{512}x^3$	B1ft
		(1)
(c)(ii)	$"2"+"2"+"-\frac{25}{64}"x^2+"-\frac{25}{64}"x^2=4-\frac{25}{32}x^2$	M1A1
		(2)
		(9 marks)

(b)

B1: $\lvert x \rvert \leqslant \frac{4}{5}$ or $\lvert x \rvert < \frac{4}{5}$ or accept strict or non-strict inequalities at either end e.g.

$-\frac{4}{5}<x<\frac{4}{5}$, $-\frac{4}{5}\leqslant x<\frac{4}{5}$, $-\frac{4}{5}<x\leqslant\frac{4}{5}$, $\frac{4}{5}>x>-\frac{4}{5}$, $x>-\frac{4}{5}$ and $x<\frac{4}{5}$

or interval notation e.g. $\left(-\frac{4}{5},\frac{4}{5}\right)$, $\left[-\frac{4}{5},\frac{4}{5}\right]$, $\left[-\frac{4}{5},\frac{4}{5}\right)$, $\left(-\frac{4}{5},\frac{4}{5}\right]$

(c)(i)

B1ft: $2-\frac{5}{4}x-\frac{25}{64}x^2-\frac{125}{512}x^3$ (follow through their (a))

For their $A+Bx+Cx^2+Dx^3$ in part (a) they must obtain $A-Bx+Cx^2-Dx^3$ where A, B, C and D are non-zero for this mark even if they make a re-start.

(c)(ii)

M1: Adds their $A+Bx+Cx^2+Dx^3$ from part (a) to $A-Bx+Cx^2-Dx^3$ to give $a+bx^2$ or obtains values for a and b.

May be implied by their answer/values if no incorrect working is seen.

A1: Cao and **cso** $4-\frac{25}{32}x^2$ o.e. e.g. $4-\frac{50}{64}x^2$ Allow $4+-\frac{25}{32}x^2$

This must follow fully correct work in (a) and (c).

Apply isw once the correct answer is seen.

Question Number	Scheme	Marks
5	$4y^2 \to 8y\frac{dy}{dx}$	M1
	$e^{xy} \to \left(y + x\frac{dy}{dx}\right)e^{xy}$ or e.g. $e^{xy} \to ye^{xy} + x\frac{dy}{dx}e^{xy}$	M1
	$2 + 8y\frac{dy}{dx} = 3\left(y + x\frac{dy}{dx}\right)e^{xy}$	A1
	$2 = 3\times\frac{9}{2}\times\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{4}{27}$	M1
	$y = \frac{4}{27}\left(x - \frac{9}{2}\right) \Rightarrow 4x - 27y - 18 = 0$	**d**M1A1
		(6)
		(6 marks)

M1: Differentiates $4y^2$ to obtain $...y\frac{dy}{dx}$

M1: Differentiates $3e^{xy}$ to obtain $...\left(y + x\frac{dy}{dx}\right)e^{xy}$ or equivalent.

Do not condone missing brackets unless they are implied by subsequent work.

A1: Fully correct differentiation $2 + 8y\frac{dy}{dx} = 3\left(y + x\frac{dy}{dx}\right)e^{xy}$ or equivalent.

Condone e.g. $\frac{dy}{dx} = 2 + 8y\frac{dy}{dx} = 3\left(y + x\frac{dy}{dx}\right)e^{xy}$ where the "$\frac{dy}{dx} =$" is their intention to differentiate.

M1: Substitutes $x = \frac{9}{2}$ and $y = 0$ and proceeds to find a value for $\frac{dy}{dx}$.

If no substitution is seen then their value must be correct for their $\frac{dy}{dx}$.

Must be a constant but may be in terms of e or may be a decimal.

Note that they may attempt to make $\frac{dy}{dx}$ the subject first e.g. $\frac{dy}{dx} = \frac{3ye^{xy} - 2}{8y - 3xe^{xy}}$

You do not need to be concerned with the details of the rearrangement.

It is dependent on having **exactly two** $\frac{dy}{dx}$ terms so if they had e.g.

$\frac{dy}{dx} = 2 + 8y\frac{dy}{dx} = 3\left(y + x\frac{dy}{dx}\right)e^{xy}$ and use all 3 $\frac{dy}{dx}$ terms, this scores M0

dM1: Uses their gradient at $\left(\frac{9}{2}, 0\right)$ with $x = \frac{9}{2}$ and $y = 0$ with the values correctly placed, in a correct straight line method for the tangent at P. Their gradient must be non-zero and must exist so do not allow methods leading to e.g. $\frac{\alpha}{0}$ where α is then used for the gradient.

If they use $y = mx + c$, they must reach as far as a value for c.

It is dependent on the previous method mark.

A1: **cso** $4x - 27y - 18 = 0$ or any integer multiple of this equation.

Note that incorrect differentiation can lead to the correct gradient and the correct answer so this mark must follow **correct work and all previous marks must be awarded.**

Question Number	Scheme	Marks
6(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}=6\cos t\times\frac{1}{-8\sin t}=k\frac{\cos t}{\sin t}$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{3}{4}\cot\left(\frac{\pi}{3}\right)=-\frac{\sqrt{3}}{4}$	**d**M1A1
		(3)

Mark (a) and (b) together.

(a)

M1: Attempts to differentiate both parametric equations to obtain $\frac{\mathrm{d}x}{\mathrm{d}t}=...\sin t$ and $\frac{\mathrm{d}y}{\mathrm{d}t}=...\cos t$ and attempts $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}\times\frac{\mathrm{d}t}{\mathrm{d}x}$ or equivalent e.g. $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}\div\frac{\mathrm{d}x}{\mathrm{d}t}$

Note that this may be done numerically e.g.

$$\frac{\mathrm{d}x}{\mathrm{d}t}=-8\sin t=-4\sqrt{3},\ \frac{\mathrm{d}y}{\mathrm{d}t}=6\cos t=3\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=3\times\frac{1}{-4\sqrt{3}}$$

dM1: Substitutes in $t=\frac{\pi}{3}$ and proceeds to find a value for $\frac{\mathrm{d}y}{\mathrm{d}x}$ which may be implied.

Depends on the previous mark.

A1: $-\frac{\sqrt{3}}{4}$ **from fully correct work** or any exact equivalent e.g. $-\frac{3}{4\sqrt{3}}$.

Isw once a correct answer is seen.

(b)	$\left(6,\ 3\sqrt{3}-3\right)$	B1B1
	$-\frac{\sqrt{3}}{4} \to \frac{4}{\sqrt{3}}$	B1ft
	$y-3\sqrt{3}+3=\frac{4}{\sqrt{3}}(x-6)$ or $y=\frac{4}{\sqrt{3}}x+c \Rightarrow -3\sqrt{3}+3=\frac{4}{\sqrt{3}}\times 6+c \Rightarrow c=\left(-3-5\sqrt{3}\right)$	M1
	When $y=0$ $-3\sqrt{3}+3=\frac{4}{\sqrt{3}}(x-6) \Rightarrow x=...$	**d**M1
	$\left(\frac{15+3\sqrt{3}}{4},\ 0\right)$	A1
		(6)

(b)

B1: One of $x=6,\ y=3\sqrt{3}-3$

B1: For both coordinates correct for *P*: $x=6,\ y=3\sqrt{3}-3$ or e.g. $\left(6,\ 3\sqrt{3}-3\right)$

Note that the above B marks may be implied and may be seen embedded in their attempt at the equation of normal and can be scored if seen anywhere in their solution.

B1ft: Gradient of the normal to *C* at *P* is $\frac{4}{\sqrt{3}}$ oe e.g. $\frac{4\sqrt{3}}{3}$ (follow through from (a)) and can be scored anywhere in their solution.

M1: Attempts the equation of the normal with values correctly placed using

- their coordinates for *P* which have come from substituting $t=\frac{\pi}{3}$ into the parametric equations
- a changed gradient from that found in (a)

If using $y = mx + c$ they must proceed to find a value for c.
If candidates use *P* in terms of t e.g. $x=8\cos t+2$ and $y=6\sin t-3$ in their normal equation then this mark will only score at the point they substitute $t=\frac{\pi}{3}$

The first 2 B marks may also be implied if the x and y coordinates can clearly be implied or seen as $x = 6$ and/or $y=3\sqrt{3}-3$ you may need to check.

dM1: Substitutes $y=0$ into their equation of the normal and proceeds to find a value for x.
It is dependent on the previous method mark.
Note that attempts to solve $6\sin t-3=0$ score M0

A1: Correct coordinates or correct value of x $\left(\frac{15+3\sqrt{3}}{4},\ 0\right)$ or $x=\frac{15+3\sqrt{3}}{4}$ o.e. e.g.

$x=\frac{9+15\sqrt{3}}{4\sqrt{3}},\ \frac{9\sqrt{3}+45}{12},\ \frac{3}{4}\left(5+\sqrt{3}\right)$ and isw once a correct answer is seen.

Allow unsimplified e.g. $x=\left(5\sqrt{3}+3\right)\div\frac{4\sqrt{3}}{3}$

(c)	$x = 8\cos t + 2 \Rightarrow \cos t = \dfrac{x-2}{8}$ $\quad y = 6\sin t - 3 \Rightarrow \sin t = \dfrac{y+3}{6}$	M1
	$\dfrac{(x-2)^2}{64} + \dfrac{(y+3)^2}{36} = 1$	A1
		(2)
		(11 marks)

(c)

M1: Attempts to rearrange either of the parametric equations to obtain $\cos t$ or $\cos^2 t$ in terms of x or $\sin t$ or $\sin^2 t$ in terms of y which may be implied by their working.
You can condone slips as long as the intention is clear e.g.

$$x = 8\cos t + 2 \Rightarrow x^2 = 8\cos^2 t + 4 \Rightarrow \cos^2 t = \frac{x^2 - 4}{8}$$

Also condone an attempt to obtain $\cos t$ or $\cos^2 t$ in terms of x or $\sin t$ or $\sin^2 t$ in terms of y if they have errors in trig identities but the intention is clear e.g. $\sin^2 t = 1 - \dfrac{x-2}{8}$

A1: $\dfrac{(x-2)^2}{64} + \dfrac{(y+3)^2}{36} = 1$ o.e. e.g. $y = 6\sqrt{1 - \left(\dfrac{x-2}{8}\right)^2} - 3$ or e.g. $y = \pm 6\sqrt{1 - \left(\dfrac{x-2}{8}\right)^2} - 3$

There will be many different alternatives. Once a correct equation is seen, award the A mark and apply isw.

Question Number	Scheme	Marks
7(a)	$\frac{\mathrm{d}u}{\mathrm{d}x}=\mathrm{e}^x$	B1
	$\int \frac{24\mathrm{e}^x-2}{\left(2\mathrm{e}^x-1\right)^2}\,\mathrm{d}x=\int \frac{24\left(u+\frac{1}{2}\right)-2}{(2u)^2}\times\frac{\mathrm{d}u}{\left(u+\frac{1}{2}\right)}$	M1
	$=\int \frac{24u+12-2}{\left(2u\right)^2}\times\frac{2}{2u+1}\mathrm{d}u=\int \frac{48u+20}{4u^2(2u+1)}\,\mathrm{d}u=\int \frac{12u+5}{u^2(2u+1)}\,\mathrm{d}u$	A1
	$x=\ln 3\rightarrow a=\frac{5}{2},\quad x=\ln 8\rightarrow b=\frac{15}{2}$	B1
		(4)

(a) **Note that apart from the limits, this is a given answer so check the work carefully.**

B1: Any correct equation connecting dx and du e.g.

$$\frac{\mathrm{d}u}{\mathrm{d}x}=\mathrm{e}^x,\ \frac{\mathrm{d}x}{\mathrm{d}u}=\frac{1}{\mathrm{e}^x},\ \mathrm{d}u=\mathrm{e}^x\mathrm{d}x,\ 1=\mathrm{e}^x\frac{\mathrm{d}x}{\mathrm{d}u},\ \mathrm{d}x=\frac{\mathrm{d}u}{\mathrm{e}^x},\ \mathrm{d}x=\frac{\mathrm{d}u}{u+\frac{1}{2}}\ \text{etc.}$$

M1: A full attempt to substitute $u=\mathrm{e}^x-\frac{1}{2}$ proceeding to an expression in terms of u only (which is not the given answer). Condone slips such as a poor rearrangement of $\frac{\mathrm{d}u}{\mathrm{d}x}=\mathrm{e}^x$ but it must include an attempt to replace dx with a function of u.

A1: Achieves the given integral with sufficient and no incorrect working seen including invisible brackets. You can ignore any limits for this mark.
There must be at least one intermediate line of working following the stage at which they obtain their integral totally in terms of u. This may be seen in "side-working".
You can condone e.g. a missing "du" in the working as long as it is present in the final answer.

B1: Correct limits seen at any stage and they do not need to be seen attached to the integral but if there is any contradiction, the limits attached to the integral take precedence.

Note that an <u>example</u> of minimum acceptable work for the first 3 marks in (a) could be:

$$\frac{\mathrm{d}u}{\mathrm{d}x}=\mathrm{e}^x,\ \int \frac{24\mathrm{e}^x-2}{\left(2\mathrm{e}^x-1\right)^2}\,\mathrm{d}x=\int \frac{24\left(u+\frac{1}{2}\right)-2}{(2u)^2}\times\frac{\mathrm{d}u}{\left(u+\frac{1}{2}\right)}=\int \frac{24u+10}{(2u)^2}\times\frac{\mathrm{d}u}{u+\frac{1}{2}}=\int \frac{12u+5}{u^2(2u+1)}\,\mathrm{d}u$$

(b)	$12u+5 \equiv A(2u+1)+Bu(2u+1)+Cu^2 \Rightarrow A=\ldots \text{or } B=\ldots \text{or } C=\ldots$ or e.g. $12u+5 \equiv (2B+C)u^2+(2A+B)u+A \Rightarrow A=\ldots \text{or } B=\ldots \text{or } C=\ldots$	M1
	2 of: $A=5,\ B=2\ ,C=-4$	A1
	$\dfrac{5}{u^2}+\dfrac{2}{u}-\dfrac{4}{(2u+1)}$	A1
		(3)

(b)

M1: Attempts to find at least one of the values for *A*, *B* and *C* by multiplying up and substituting values or by multiplying up and comparing coefficients.
When multiplying up must have at least one of the terms on the rhs correct for their lhs.

A1: Two of $A=5,\ B=2,\ C=-4$

A1: Correct **fractions**: $\dfrac{5}{u^2}+\dfrac{2}{u}-\dfrac{4}{(2u+1)}$ oe. Allow $\dfrac{5}{u^2}+\dfrac{2}{u}+-\dfrac{4}{(2u+1)}$
If the correct fractions are not seen in part (b), allow this mark to score if the correct fractions are seen in part (c).

(c)	$\displaystyle\int \frac{5}{u^2}+\frac{2}{u}-\frac{4}{(2u+1)}\,\mathrm{d}u = -5u^{-1}+2\ln u-2\ln(2u+1)\ \ (+c)$	M1A1ft
	$\left[-5u^{-1}+2\ln u-2\ln(2u+1)\right]_{2.5}^{7.5}$ $=\left(-5\times7.5^{-1}+2\ln7.5-2\ln(16)\right)-\left(-5\times2.5^{-1}+2\ln2.5-2\ln(6)\right)$ or $\left[-5\left(\mathrm{e}^x-\frac{1}{2}\right)^{-1}+2\ln\left(\mathrm{e}^x-\frac{1}{2}\right)-2\ln(2\left(\mathrm{e}^x-\frac{1}{2}\right)+1)\right]_{\ln3}^{\ln8}$ $=\left(-5\times7.5^{-1}+2\ln7.5-2\ln(16)\right)-\left(-5\times2.5^{-1}+2\ln2.5-2\ln(6)\right)$	**d**M1
	$\dfrac{4}{3}+\ln\left(\dfrac{81}{64}\right)$	A1
		(4)
		(11 marks)

(c)

M1: Integrates either $\dfrac{"2"}{u}\to\ldots\ln ku$ **or** $\dfrac{"4"}{(2u+1)}\to\ldots\ln k(2u+1)$

A1ft: $-5u^{-1}+2\ln u-2\ln(2u+1)$ oe following through on their non-zero *A*, *B* and *C* from part (b).
Allow unsimplified.

dM1: Substitutes in their changed limits from part (a) or transforms back in terms of *x* i.e. ln3 and ln8 and subtracts either way round.
Depends on the previous method mark.

A1: $\dfrac{4}{3}+\ln\left(\dfrac{81}{64}\right)$ or equivalent in the required form i.e. $p+\ln q$

Question Number	Scheme	Marks
8(a)(i)	$\pm\left(\left(3\mathbf{i}+4\mathbf{j}-6\mathbf{k}\right)-\left(2\mathbf{i}-\mathbf{j}+5\mathbf{k}\right)\right)$	M1
	$=\mathbf{i}+5\mathbf{j}-11\mathbf{k}$	A1
(ii)	e.g. $\mathbf{r}=2\mathbf{i}-\mathbf{j}+5\mathbf{k}+\lambda\left(\mathbf{i}+5\mathbf{j}-11\mathbf{k}\right)$ or $\mathbf{r}=3\mathbf{i}+4\mathbf{j}-6\mathbf{k}+\lambda\left(\mathbf{i}+5\mathbf{j}-11\mathbf{k}\right)$ o.e.	A1ft
		(3)

(a)

(i)

M1: Attempts to find the difference between the two vectors either way round.
The expression e.g. $\left(\left(3\mathbf{i}+4\mathbf{j}-6\mathbf{k}\right)-\left(2\mathbf{i}-\mathbf{j}+5\mathbf{k}\right)\right)$ is sufficient.
If no working is shown, it may be implied by 2 correct components of $\pm\left(\mathbf{i}+5\mathbf{j}-11\mathbf{k}\right)$ or e.g. $\pm(1, 5, -11)$

A1: $\mathbf{i}+5\mathbf{j}-11\mathbf{k}$ o.e. e.g. $\begin{pmatrix}1\\5\\-11\end{pmatrix}$ and condone $\begin{matrix}1\\5\\-11\end{matrix}$ but **not** $\begin{pmatrix}\mathbf{i}\\5\mathbf{j}\\-11\mathbf{k}\end{pmatrix}$ and not (1, 5,–11)
But allow this mark once a correct form is seen.

(ii)

A1ft: Any correct vector equation for the line **following through** their $\overrightarrow{AB}$ **but must follow M1.**
Must include "**r** =" (not e.g. "*l* =") and allow any parameter.

e.g. $\mathbf{r}=\begin{pmatrix}2\\-1\\5\end{pmatrix}+\lambda\begin{pmatrix}1\\5\\-11\end{pmatrix}$ or e.g. $\mathbf{r}=\begin{pmatrix}3\\4\\-6\end{pmatrix}+\mu\begin{pmatrix}-1\\-5\\11\end{pmatrix}$

Do **not** allow e.g. $\mathbf{r}=\begin{pmatrix}2\\-1\\5\end{pmatrix}+t\begin{pmatrix}\mathbf{i}\\5\mathbf{j}\\-11\mathbf{k}\end{pmatrix}$ **unless** the notation $\begin{pmatrix}\mathbf{i}\\5\mathbf{j}\\-11\mathbf{k}\end{pmatrix}$ has already been penalised on the previous A mark and this is the only error in their notation.

Do **not** allow e.g. $l=\begin{pmatrix}2\\-1\\5\end{pmatrix}+\lambda\begin{pmatrix}1\\5\\-11\end{pmatrix}$ or e.g. $l:\begin{pmatrix}2\\-1\\5\end{pmatrix}+\lambda\begin{pmatrix}1\\5\\-11\end{pmatrix}$

But condone $l:\mathbf{r}=\begin{pmatrix}2\\-1\\5\end{pmatrix}+\lambda\begin{pmatrix}1\\5\\-11\end{pmatrix}$

(b)	$\overrightarrow{CA} = (2-p)\mathbf{i} - 5\mathbf{j} + 6\mathbf{k} \qquad \overrightarrow{CB} = (3-p)\mathbf{i} - 5\mathbf{k}$	M1
	$(2-p)(3-p) - 30 = 0$	**d**M1
	$\Rightarrow 6 - 5p + p^2 - 30 = 0 \Rightarrow p^2 - 5p - 24 = 0 \Rightarrow p = ...$	**dd**M1
	$p = -3, \quad p = 8$	A1
		(4)
(c)	$\left\lvert\overrightarrow{CA}\right\rvert = \sqrt{(2-8)^2 + (-5)^2 + 6^2}$ or $\left\lvert\overrightarrow{CB}\right\rvert = \sqrt{(3-8)^2 + (-5)^2}$	M1
	$\left\lvert\overrightarrow{CA}\right\rvert = \sqrt{97}$ or $\left\lvert\overrightarrow{CB}\right\rvert = \sqrt{50}$	A1
	Area of triangle = $\frac{1}{2} \times \left\lvert\overrightarrow{CA}\right\rvert \times \left\lvert\overrightarrow{CB}\right\rvert = \frac{1}{2} \times \sqrt{50} \times \sqrt{97} =$ awrt 34.8 units^2 or e.g. $\frac{1}{2} \times \left\lvert\overrightarrow{CA}\right\rvert \times \left\lvert\overrightarrow{AB}\right\rvert \sin BAC = \frac{1}{2} \times \sqrt{50} \times \sqrt{147} \sin\left(\cos^{-1}\frac{\sqrt{50}}{\sqrt{147}}\right) =$ awrt 34.8 units^2	**d**M1A1
		(4)
		(11 marks)

(b) **Allow all marks in (b) for correct work using** $\pm\overrightarrow{CA}$ and/or $\pm\overrightarrow{CB}$

M1: Attempts to find either $\pm\overrightarrow{CA}$ or $\pm\overrightarrow{CB}$ using subtraction.
If no working is shown, it may be implied by 2 correct components.

dM1: Depends on the previous method mark.
Attempts both $\pm\overrightarrow{CA}$ **and** $\pm\overrightarrow{CB}$ using subtraction and attempts the scalar product between their $\pm\overrightarrow{CA}$ and their $\pm\overrightarrow{CB}$ **and** sets = 0 to obtain $\pm(2-p)\times\pm(3-p)\pm k = 0$ where $k \neq 0$

ddM1: Depends on both previous method marks.
Multiplies out, collects terms and attempts to solve their three term quadratic (usual rules apply – see general guidance). May be implied by correct roots for their 3TQ.
Do not condone work where they lose or ignore their *k*.

A1: Correct values $p = -3, \quad p = 8$

(c)

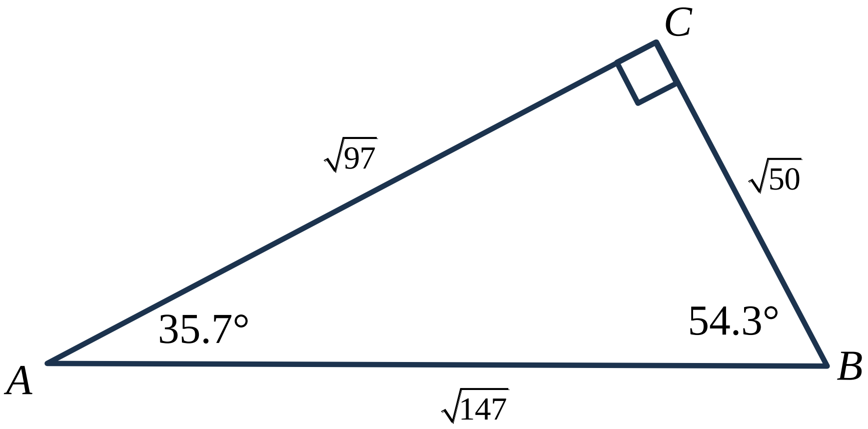
M1: Uses at least one of their values of *p* from part (b) (which may be negative) and attempts $\left\lvert\overrightarrow{CA}\right\rvert$ **or** $\left\lvert\overrightarrow{CB}\right\rvert$ using Pythagoras. May be implied.

A1: For $\left\lvert\overrightarrow{CA}\right\rvert = \sqrt{97}$ or $\left\lvert\overrightarrow{CB}\right\rvert = \sqrt{50}$ seen or implied.

dM1: Depends on the previous method mark.
Attempts to find the area of the triangle using a correct method e.g. $\frac{1}{2} \times$ their $\left\lvert\overrightarrow{CA}\right\rvert \times$ their $\left\lvert\overrightarrow{CB}\right\rvert$
It is dependent on the previous method mark and it is dependent on having used a **positive** value for *p*.
If they use a less direct method e.g. $\frac{1}{2} \times \left\lvert\overrightarrow{CA}\right\rvert \times \left\lvert\overrightarrow{AB}\right\rvert \sin BAC$ then it must be a correct method, including an attempt to find $\left\lvert\overrightarrow{AB}\right\rvert$ using Pythagoras.

A1: awrt 34.8 This answer and no others. Do not allow for exact answers.
If they use the other value of *p* and give another area which is not rejected, score A0

For reference:



Note that other methods may be possible in (c) for the area e.g. using a vector product:

$$p = 8 \Rightarrow \overrightarrow{CA} = -6\mathbf{i} - 5\mathbf{j} + 6\mathbf{k}, \quad \overrightarrow{CB} = -5\mathbf{i} - 5\mathbf{k}$$

$$\overrightarrow{CA} \times \overrightarrow{CB} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -6 & -5 & 6 \\ -5 & 0 & -5 \end{vmatrix} = \begin{pmatrix} 25 \\ -60 \\ -25 \end{pmatrix}$$

$$\text{Area of triangle} = \frac{1}{2}\sqrt{25^2 + 60^2 + 25^2} = 34.8$$

Score as:

M1: Attempts the vector product between 2 appropriate vectors e.g. $\overrightarrow{CA}$ and $\overrightarrow{CB}$
May be implied by 2 correct components.

A1: Correct vector

dM1: Attempts $\frac{1}{2} \times$ the magnitude of their vector

A1: For awrt 34.8

Question Number	Scheme	Marks
9(a)	Volume $=144\pi\int_0^{20}\frac{2x+3}{x^2+3x+5}\,\mathrm{d}x$ or e.g. $\pi\int_0^{20}\left(12\left(\frac{2x+3}{x^2+3x+5}\right)^{\frac{1}{2}}\right)^2\mathrm{d}x$	B1
	$144\pi\left[\ln\left(x^2+3x+5\right)\right]_0^{20}$	M1
	$=144\pi\left(\ln 465-\ln 5\right)=144\pi\ln 93\,(\text{units}^3)$	**d**M1A1
		(4)

(a)

B1: Correct integral for the volume of R which may be simplified or unsimplified.
The 144 (or 12^2) and the π and the limits 0 and 20 must be present or implied by subsequent work. Condone the omission of "dx".
$2\pi\times144\int_0^{20}\frac{2x+3}{x^2+3x+5}\,\mathrm{d}x$ scores B0

M1: Integrates $\frac{2x+3}{x^2+3x+5}$ to obtain $...\ln\left(x^2+3x+5\right)$
Condone missing brackets e.g. $...\ln x^2+3x+5$
Condone poor notation e.g. $144\pi\int\frac{2x+3}{x^2+3x+5}\mathrm{d}x=\int144\pi\left[\ln\left(x^2+3x+5\right)\right]_0^{20}\mathrm{d}x$
This may be implied by substitution e.g. $u=x^2+3x\Rightarrow\int\frac{2x+3}{x^2+3x+5}\mathrm{d}x=...\ln\left(u+5\right)$
or $u=x^2+3x+5\Rightarrow\int\frac{2x+3}{x^2+3x+5}\mathrm{d}x=...\ln u$

dM1: Substitutes 20 **and** 0 into their expression of the form $...\ln\left(x^2+3x+5\right)$, subtracts either way round and proceeds to find an **exact** expression for the volume e.g. $144\pi\left(\ln 465-\ln 5\right)$
If they use substitution, the limits must be **correct for their substitution** e.g.
$u=x^2+3x\Rightarrow u=0,\ 460$ or e.g. $u=x^2+3x+5\Rightarrow u=5,\ 465$
Do not allow e.g. $144\pi\left(\ln 465-0\right)$ i.e. no evidence of the use of the lower limit.
Depends on the previous mark.

A1: $144\pi\ln 93$ (condone lack of units)

Note if π is missing throughout in (a), a maximum of B0M1dM1A0 is possible.

Example working if a substitution is used e.g.

$$u=x^2+3x+5\Rightarrow\frac{\mathrm{d}u}{\mathrm{d}x}=2x+3\Rightarrow144\pi\int\frac{2x+3}{u}\,\frac{\mathrm{d}x}{2x+3}=144\pi\left[\ln u\right]_5^{465}$$

$$144\pi\left(\ln 465-\ln 5\right)=144\pi\ln 93$$

(b)	$144\pi\left[\ln\left(x^2+3x+5\right)\right]_p^{20}=360\pi$ $\Rightarrow 144\pi\left(\ln 465-\ln\left(p^2+3p+5\right)\right)=360\pi$ **or e.g.** Let $p=20-a$: $144\pi\left[\ln\left(x^2+3x+5\right)\right]_{20-a}^{20}=360\pi$ $\Rightarrow 144\pi\left(\ln 465-\ln\left(\left(20-a\right)^2+3\left(20-a\right)+5\right)\right)=360\pi$ **or e.g.** $144\pi\left[\ln\left(x^2+3x+5\right)\right]_0^{20-a}=144\pi\ln 93-360\pi$ $\Rightarrow 144\pi\left(\ln\left(\left(20-a\right)^2+3\left(20-a\right)+5\right)-\ln 5\right)=144\pi\ln 93-360\pi$ **or e.g.** $144\pi\left[\ln\left(x^2+3x+5\right)\right]_0^{p}=144\pi\ln 93-360\pi$ $\Rightarrow 144\pi\left(\ln\left(p^2+3p+5\right)-\ln 5\right)=144\pi\ln 93-360\pi$	M1
	$\frac{5}{2}=\ln\left(\frac{465}{p^2+3p+5}\right)\Rightarrow e^{\frac{5}{2}}=\frac{465}{p^2+3p+5}$ $\Rightarrow p^2+3p+5-465e^{-\frac{5}{2}}=0\Rightarrow p=...$ **or e.g.** $\ln\left(\frac{a^2-43a+465}{5}\right)=\frac{144\pi\ln 93-360\pi}{144\pi}$ $\Rightarrow \frac{a^2-43a+465}{5}=e^{\left(\frac{144\ln 93-360}{144}\right)}\Rightarrow a^2-43a+465-5e^{\left(\frac{144\ln 93-360}{144}\right)}=0\Rightarrow a=...$ **or e.g.** $\ln\left(\frac{p^2+3p+5}{5}\right)=\frac{144\pi\ln 93-360\pi}{144\pi}$ $\Rightarrow \frac{p^2+3p+5}{5}=e^{\left(\frac{144\ln 93-360}{144}\right)}\Rightarrow p^2+3p+5-5e^{\left(\frac{144\ln 93-360}{144}\right)}=0\Rightarrow p=...$	**d**M1
	$a=$ awrt 15.5	A1
		(3)
		(7 marks)

(b)

M1: Sets an expression of the form $...\left[\ln\left(x^2+3x+5\right)\right]_p^{20}$ equal to 360π and substitutes in the limits 20 and p (or any other letter – condone a) and subtracts either way round.
Alternatively sets an expression of the form $...\left[\ln\left(x^2+3x+5\right)\right]_0^{20-a}$ or e.g. $...\left[\ln\left(x^2+3x+5\right)\right]_0^{p}$ (or any other letter – condone a) equal to 360π – their answer to part (a) and substitutes in the limits 20 – a and 0 or e.g. 0 and p and subtracts either way round.
For this mark, their method, if executed correctly would either lead directly to the value of a or would enable a to be found by e.g. a = 20 – their value.

dM1: **Depends on the previous mark.**
Removes lns, rearranges to form a three term quadratic and attempts to solve, proceeding to a value for p. In the alternative method this mark is scored for proceeding to find a value for

A1:	*a*. The usual rules apply for solving their 3TQ although this is most likely to be done on a calculator so you may need to check. cao awrt 15.5 with or without units Note that if they find "*p*" first then then need to find 20 – "*p*" to obtain the correct value of *a*. For reference, "*p*" = 4.45143…..

Examples in (b) that score no marks:

$$144\pi\left[\ln\left(x^2+3x+5\right)\right]_0^p=360\pi$$

$$144\pi\left[\ln u\right]_5^{p^2+3p+5}=360\pi$$

(Having substituted $u=x^2+3x+5$ in (a))

Example of correct working if a substitution is used e.g. $u=x^2+3x+5$

Let $p=20-a$: $144\pi\left[\ln u\right]_{p^2+3p+5}^{465}=360\pi$

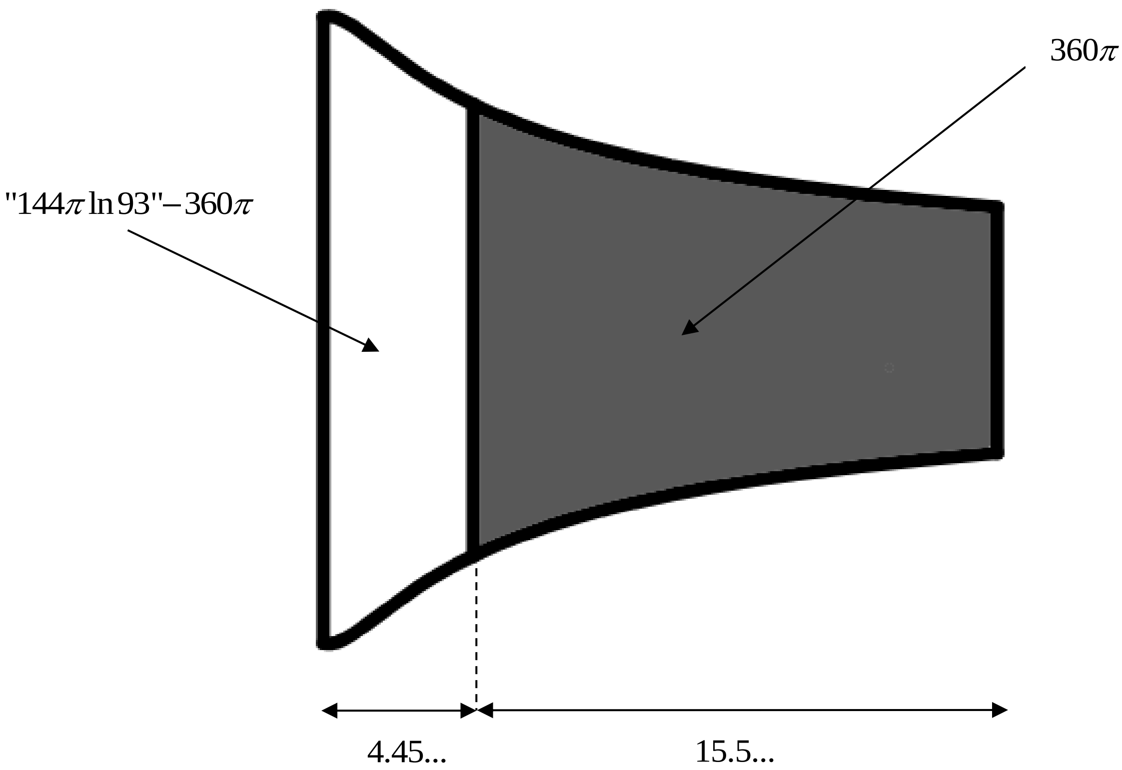
$$\Rightarrow\frac{5}{2}=\ln\left(\frac{465}{p^2+3p+5}\right)\Rightarrow e^{\frac{5}{2}}=\frac{465}{p^2+3p+5}\text{ etc.}$$

or e.g.

$$144\pi\left[\ln u\right]_5^{(20-a)^2+3(20-a)+5}=144\pi\ln 93-360\pi$$

$$\Rightarrow 144\pi\left(\ln\left((20-a)^2+3(20-a)+5\right)-\ln 5\right)=144\pi\ln 93-360\pi\text{ etc.}$$

<u>For reference:</u>



Question 10

Note that there are many different ways to answer this question.

General Guidance:

B1: For setting up the correct contradiction.
It must include a word/words such as "assume" or "let" or "if" and $p^2 - 4q + 2 = 0$
As a minimum accept just "assume/let/if $p^2 - 4q + 2 = 0$"

M1: Attempts to make p^2 the subject.

A1: Makes a correct deduction following correct work.

dM1: Continues the argument/algebra that will lead to a contradiction.

A1: For a fully correct proof.
Requires

- all previous marks scored
- no incorrect assumptions/statements
- correct calculations/algebra
- a reason: e.g. $2q - 2k^2 = 1$ is not true
- E.g. hence $p^2 - 4q + 2 \neq 0$, hence proven, QED, etc. but not just "contradiction"

Question Number	Scheme	Marks
10 Main	Assume that $p^2-4q+2=0$	B1
	$p^2=4q-2$	M1
	$p^2=4q-2\left(=2(2q-1)\right)$ So p^2 is even so p is even oe e.g. so $p=2k$	A1
	If $p=2k$ then e.g. $4k^2=4q-2$ or e.g. $4k^2=4q-2\Rightarrow q=k^2+\frac{1}{2}$ or e.g. $q=\frac{2k^2+1}{2}$ or e.g. $4k^2=4q-2\Rightarrow k^2=q-\frac{1}{2}$ or e.g. $k^2=\frac{2q-1}{2}$ or e.g. Let $4k^2=4q-2\Rightarrow 2q-2k^2=1$	**d**M1
	$4k^2$ is a multiple of 4 and $4q-2$ isn't a multiple of 4 or e.g. $q=k^2+\frac{1}{2}$ which is not an integer or e.g. $k^2=q-\frac{1}{2}$ which is not an integer or e.g. $2q-2k^2=1$ which is not possible So $p^2-4q+2\neq 0$	A1
		(5 marks)

Way 1

B1: For setting up the correct contradiction.
It must include a word/words such as "assume" or "let" or "if" **and** $p^2-4q+2=0$
As a minimum accept just "assume/let/if" $p^2-4q+2=0$"

M1: Rearranges $p^2-4q+2=0$ to obtain $p^2=\pm 4q\pm 2$

A1: Correct deduction for p following **correct work**.
It is not necessary to factorise $4q-2$ but if they do it must be correct.
It requires e.g. both p^2 is even **and** so p is even oe e.g. so $p=2k$ or any other letter but not $p = 2q$ or $p = 2p$

dM1: **Depends on the previous method mark.**
For attempting to use the fact that p is even to obtain a contradiction e.g. uses that as p is even $p=2k$ and substitutes into the equation to obtain e.g.
$4k^2=4q-2$ or $q=k^2+\frac{1}{2}$ or $k^2=q-\frac{1}{2}$ or $2q-2k^2=1$

A1: Fully correct work with all previous marks scored that includes
- An explanation why there is a contradiction
- A (minimal) conclusion e.g. "so $p^2-4q+2\neq 0$", QED, "proven", ✓

Must follow all previous marks and there must be no incorrect statements/algebra in their proof.

An odds/evens approach:

Alt		
	Assume that $p^2-4q+2=0$	B1
	Evens: Let $p=2k$, $(2k)^2-4q+2=0$ **or** Odds: Let $p=2k+1$, $(2k+1)^2-4q+2=0$	M1
	$4q=4k^2+2=4\left(k^2+\frac{1}{2}\right)$ which is not a multiple of 4 oe e.g. $4k^2+2$ is 2 more than a multiple of 4 **or** $4q=4k^2+4k+3=4\left(k^2+k+\frac{3}{4}\right)$ which is not a multiple of 4 oe e.g. $4k^2+4k+3$ is 3 more than a multiple of 4	A1
	Evens: Let $p=2k$, $(2k)^2-4q+2=0$ **and** Odds: Let $p=2k+1$, $(2k+1)^2-4q+2=0$	**d**M1
	$4q=4k^2+2=4\left(k^2+\frac{1}{2}\right)$ which is not a multiple of 4 oe e.g. $4k^2+2$ is 2 more than a multiple of 4 **and** $4q=4k^2+4k+3=4\left(k^2+k+\frac{3}{4}\right)$ which is not a multiple of 4 oe e.g. $4k^2+4k+3$ is 3 more than a multiple of 4 $p^2-4q+2\neq 0$	A1

Alt

B1: For setting up the correct contradiction.
It must include a word/words such as "assume" or "let" or "if" and $p^2-4q+2=0$
As a minimum accept just "assume/let/if" $p^2-4q+2=0$"

M1: Attempts $p^2-4q+2=0$ with p even e.g. p = 2k or equivalent.
or attempts $p^2-4q+2=0$ with p odd e.g. p = 2k + 1 or equivalent e.g. 2k – 1

A1: Correct algebra and deduction for odds **or** evens

dM1: Depends on the previous method mark.
Attempts $p^2-4q+2=0$ with p even e.g. p = 2k or equivalent.
and attempts $p^2-4q+2=0$ with p odd e.g. p = 2k + 1 or equivalent e.g. 2k – 1

A1: Fully correct work with all previous marks scored that includes
- Correct arguments for both odds and evens
- A (minimal) conclusion e.g. "so $p^2-4q+2\neq 0$", QED, "proven", ✓

Must follow all previous marks and there must be no incorrect statements/algebra in their proof.

Special case:

Assume that $p^2 - 4q + 2 = 0$

$$p^2 = 4q - 2$$

$$p = \sqrt{4q-2} \Rightarrow p = 2\sqrt{q - \frac{1}{2}}$$

On its own scores B1M1A0dM0A0

The B1M1 score as in the general guidelines.

To score any further marks, they would need a complete proof showing why $2\sqrt{q-\frac{1}{2}}$ cannot be an integer.

See supplementary document for some example responses and suggested marking.

If a response has a mix of approaches, mark the attempt that gives the most credit.

If you are in doubt if a particular approach deserves credit then use Review.

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