Pearson Edexcel

Mark Scheme (Results)

Summer 2025

Pearson Edexcel International Advanced Level in Pure Mathematics P4 (WMA14)
Paper 01

Summer 2025
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)

   Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN:
   - bod – benefit of doubt
   - ft – follow through
     - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC – special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp – decimal places
   - sf – significant figures
   - ∗ – The answer is printed on the paper or ag- answer given
   - $\llcorner$ or d… – The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by ‘MR’ in the body of the script.

6. If a candidate makes more than one attempt at any question:
   a) If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   b) If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \ldots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $\quad q \neq 0$ leading to $x = \ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice

Question Number	Scheme	Marks
1(a)	$2y^2-3y=7+13 \Rightarrow 2y^2-3y-20=0 \Rightarrow y=...$	M1
	$y=-\frac{5}{2}$ and $y=4$	A1
		(2)
(b)	$2y^2 \to 4y\frac{\mathrm{d}y}{\mathrm{d}x}$	B1
	$-6xy \to \pm...x\frac{\mathrm{d}y}{\mathrm{d}x}\pm...y$	M1
	$4y\frac{\mathrm{d}y}{\mathrm{d}x}-6x\frac{\mathrm{d}y}{\mathrm{d}x}-6y=14\mathrm{e}^{2x-1}$	A1
	$4(4)\frac{\mathrm{d}y}{\mathrm{d}x}-6\left(\frac{1}{2}\right)\frac{\mathrm{d}y}{\mathrm{d}x}-6(4)=14\mathrm{e}^{2\left(\frac{1}{2}\right)-1} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=...$	M1
	$y-"4"="\frac{38}{13}"\left(x-\frac{1}{2}\right)$	M1
	$38x-13y+33=0$	A1
		(6)
		(8 marks)

(a)

M1: Attempts all of the following

- substitutes $x=\frac{1}{2}$ into the equation for C (condoning slips),
- collect terms and forms a 3TQ in y (terms do not need to be on one side of the equation and the '= 0' may be implied by subsequent working)
- solves their 3TQ in y using usual rules (including via a calculator) to find at least one value for y. If a calculator is used, the values must be correct for their 3TQ

A1: Achieves both values $-\frac{5}{2}$, 4 following the award of M1

(b)

B1: $2y^2 \to 4y\frac{dy}{dx}$ o.e. For example, accept $2y^2 \to 2\times 2y\frac{dy}{dx}$

M1: Attempts the product rule on $-6xy$. Look for $\pm\alpha x\frac{dy}{dx}\pm\beta y$

Don't be concerned by the signs here unless they state and use an incorrect product rule $vu'-uv'$

A1: Fully correct differentiation $4y\frac{dy}{dx}-6x\frac{dy}{dx}-6y=14e^{2x-1}$ o.e. Allow un-simplified versions

You may see versions like $4y\,dy-6x\,dy-6y\,dx=14e^{2x-1}\,dx$ or $4yy'-6xy'-6y=14e^{2x-1}$ which is acceptable. If they write $\frac{dy}{dx}=4y\frac{dy}{dx}-6x\frac{dy}{dx}-6y-14e^{2x-1}=0$ this will be A0 unless the $\frac{dy}{dx}$ on the left subsequently disappears and is ignored.

M1: Attempts to substitute in $x=\frac{1}{2}$ with their positive y value to find a value for the gradient of the tangent. It is dependent upon having exactly two terms in $\frac{dy}{dx}$ one each from the differentiation of $2y^2$ and $6xy$. This can be implied by a correct value for their $\frac{dy}{dx}$. Allow slips if they attempt to make $\frac{dy}{dx}$ the subject before substituting in $x=\frac{1}{2}$ with their positive y value. FYI $\frac{dy}{dx}=\frac{6y+14e^{2x-1}}{4y-6x}\to\frac{dy}{dx}=\frac{24+14}{16-3}=\frac{38}{13}$

M1: Uses the value for their $\frac{dy}{dx}$ with the point $\left(\frac{1}{2},a\right)$ where a is their positive y coordinate to find the equation of the tangent. If they have two positive y values, use of either one would be OK for this mark. It must follow some attempt at differentiating the equation for C. If the form $y=mx+c$ is used the method must proceed as far as $c=\ldots$

A1: $38x-13y+33=0$ (or any integer multiple of this). The order of the terms is not important

Question Number	Scheme	Marks
2	$\frac{dV}{dt} = \pm k$	B1
	$V = \frac{4}{3}\pi r^3 \rightarrow \frac{dV}{dr} = 4\pi r^2$	B1
	$\pm k = \lambda r^2 \times \frac{dr}{dt}$	M1
	$\frac{dr}{dt} = \frac{\pm k}{4\pi r^2} \Rightarrow \frac{dr}{dt} \propto \frac{1}{r^2}$ ✓ *	A1*
		(4 marks)

B1: States or uses $\frac{dV}{dt} = k$ or $\frac{dV}{dt} = -k$ (We don't know if k is positive or negative).

Ignore any reference to units even if they are incorrect. E.g. $\frac{dV}{dt} = k \text{ cm}^3 / \text{sec}$

B1: States or uses $\frac{dV}{dr} = 4\pi r^2$ o.e.

M1: Uses a correct chain rule e.g. $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$ or equivalent with $\frac{dV}{dt} = \pm k$ and $\frac{dV}{dr} = \lambda r^2$

A1*: Completes the proof by looking for the following three elements

- achieves $\frac{dr}{dt} = \frac{\pm k}{4\pi r^2}$
- Offers a valid reason why $\frac{dr}{dt} \propto \frac{1}{r^2}$
- gives a minimal conclusion

All previous marks must have been awarded

Examples of acceptable reasons and acceptable conclusions are;

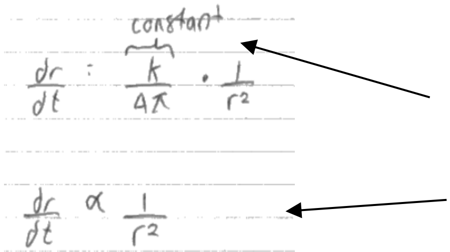
E.g. I $\frac{dr}{dt} = -\frac{k}{4\pi r^2} = -\frac{c}{r^2}$, hence proven (Note that c cannot be k)

E.g. II $\frac{dr}{dt} = \frac{k}{4\pi r^2}$, as $\frac{k}{4\pi}$ is a constant $\frac{dr}{dt} \propto \frac{1}{r^2}$

Eg. III $\frac{dr}{dt} = -\frac{k}{4\pi r^2}$ as k and π are constants, it is true

E.g. IV $\frac{dr}{dt} = -\frac{k}{4\pi r^2} = -\frac{k}{4\pi} \times \frac{1}{r^2}$ or $\left(-\frac{k}{4\pi}\right) \times \frac{1}{r^2}$ ✓

E.g.V.



Reason

Conclusion

Question Number	Scheme	Marks
3	$y\cos^2(2x)\dfrac{dy}{dx}=3\sin 2x\to\int y\ dy=\int 3\dfrac{\sin 2x}{\cos^2 2x}\ dx$	M1
	$\int y\ dy=\int 3\sec(2x)\tan(2x)\,dx$	A1
	$\dfrac{y^2}{2}=\dfrac{3}{2}\sec 2x\ (+c)$	M1A1
	$\dfrac{16}{2}=\dfrac{3}{2}\sec\left(2\dfrac{\pi}{6}\right)+c\Rightarrow c=...\ (=5)$	dM1
	$y^2=3\sec 2x+10$	A1
		(6)

Main method

M1: For an attempt to separate the variables **OR** use of $\dfrac{\sin 2x}{\cos^2 2x}\equiv\ \sec 2x\tan 2x$

For the attempt at separation of variables it is not necessary to have the integral signs but the dx and dy must be present and in the correct positions (on at least one line)

E.g. $y\cos^2(2x)\dfrac{dy}{dx}=3\sin 2x\to\int y\ dy=\int 3\dfrac{\sin 2x}{\cos^2 2x}\ dx$ or $\int y\dfrac{dy}{dx}dx=\int 3\dfrac{\sin 2x}{\cos^2 2x}\ dx$

Condone a slip on the position (or omission) of the 3

A1: Correct separation of variables **AND** use of $\dfrac{\sin 2x}{\cos^2 2x}=\ \sec 2x\tan 2x$

E.g. $\int y\ dy=\int 3\sec(2x)\tan(2x)\,dx$ o.e. with or without the integral signs

Note that $\int\dfrac{1}{3}y\ dy=\int\sec(2x)\tan(2x)\,dx$ is also correct

M1: Correct attempted integration of the $\sec 2x\tan 2x$ to reach $f(y)=\alpha\sec 2x\ (+c)$ with or without the c

A1: Correct integration.

For example, $\dfrac{y^2}{2}=\dfrac{3}{2}\sec 2x\ (+c)$ Accept with or without the c

Note that that $y^2=3\sec 2x\ (+c)$ and $\dfrac{y^2}{3}=\sec 2x\ (+c)$ are also examples of correct work.

dM1: Substitutes in $y=4$ and $x=\dfrac{\pi}{6}$ into an integrated equation of the form $f(y)=\alpha\sec 2x+c$ and proceeds to find c. It is dependent on the previous method mark. You may need to check calculations if the substitution isn't shown. You may see this attempted via $\left[\dfrac{y^2}{2}\right]_4^y=\left[\dfrac{3}{2}\sec 2x\right]_{\frac{\pi}{6}}^x$

A1: $y^2=3\sec 2x+10$ or equivalent such as $y^2=\dfrac{3}{\cos 2x}+10$ but ISW after a correct answer.

The answer is required to be in the form $y^2=$

A score of 1,0,1,1,1,1 is possible for candidates who never have a dx and/or a dy when separating the variables, but do everything else correctly.

Question Number	Scheme	Marks
3 ALT	$y\cos^2(2x)\frac{dy}{dx}=3\sin 2x \to \int y\ dy=\int 3\frac{\sin 2x}{\cos^2 2x}\ dx \to \int y\ dy = 3(\cos 2x)^n$ where $n=-1$ or -3	M1, A1
	$\frac{y^2}{2}=\frac{3}{2}(\cos 2x)^{-1}\ (+c)$	M1A1
	$\frac{16}{2}=\frac{3}{2\cos\left(\frac{\pi}{3}\right)}+c \Rightarrow c=...\ (=5)$	dM1
	$y^2=\frac{3}{\cos 2x}+10$	A1

Alt versions will integrate $\frac{\sin 2x}{\cos^2 2x}$ using substitution or reverse chain rule

The key here is realising that the integral of this is $(\cos 2x)^n$ or u^n where $u=\cos 2x$

M1**:** For an attempt to separate the variables **OR** use of $\int \frac{\sin 2x}{\cos^2 2x}dx \equiv (\cos 2x)^n$

For separation of variables look for example for $y\cos^2(2x)\frac{dy}{dx}=3\sin 2x \to \int y\ dy=\int 3\frac{\sin 2x}{\cos^2 2x}\ dx$.
condoning a slip on the position (or omission) of the 3 and omission of the integral signs

A1: Correct separation of variables **AND** use of $\int \frac{\sin 2x}{\cos^2 2x}dx \equiv (\cos 2x)^n$ where $n=-1$ or -3

If the candidate sets $u=\cos 2x$ it is for integrating $\int \frac{\sin 2x}{\cos^2 2x}dx$ to u^n where $n=-1$ or -3

M1: Correct attempted integration of the $\int \frac{\sin 2x}{\cos^2 2x}dx$ to reach $f(y)=\frac{\alpha}{\cos 2x}\ (+c)$ o.e with or without the c

A1: Correct integration. For example, $\frac{y^2}{2}=\frac{3}{2\cos 2x}\ (+c)$ o.e. Accept with or without the c

Note that that $y^2=\frac{3}{\cos 2x}\ (+c)$ and $\frac{y^2}{3}=(\cos 2x)^{-1}\ (+c)$ are also examples of correct work.

dM1: Substitutes in $y=4$ and $x=\frac{\pi}{6}$ into an integrated equation of the form $f(y)=\frac{\alpha}{\cos 2x}+c$ o.e. and proceeds to find c. It is dependent on the previous method mark.

A1: $y^2=\frac{3}{\cos 2x}+10$ or equivalent such as $y^2=\frac{3}{\cos 2x}+10$ but ISW after a correct answer.
The answer is required to be in the form $y^2=$

……..

Also note that you may attempt using integration by parts. FYI $\int 3\sin 2x\sec^2 2x\ dx=\frac{3}{2}\sin 2x\tan 2x+\frac{3}{2}\cos 2x$

1st A1: Correct separation of variables and $\int 3\sin 2x\sec^2 2x\ dx=\pm\lambda\sin 2x\tan 2x\pm\kappa\cos 2x \quad \lambda,\kappa>0$

2nd M1: $\int 3\sin 2x\sec^2 2x\ dx=\lambda\sin 2x\tan 2x\pm\kappa\cos 2x \quad \lambda,\kappa>0$

2nd A1: $\int 3\sin 2x\sec^2 2x\ dx=\frac{3}{2}\sin 2x\tan 2x+\frac{3}{2}\cos 2x$

Question Number	Scheme	Marks
4(a)	e.g. $5+17x-10x^2 = A(2x+1)(1-x)+Bx(2x+1)+Cx(1-x)$ $\Rightarrow A = ..., \ B = ..., \ C = ...$	M1
	$A=5,\ B=4,\ C=8 \Rightarrow \mathrm{f}(x) = \dfrac{5}{x}+\dfrac{4}{(1-x)}+\dfrac{8}{(2x+1)}$	A1A1
		(3)
(b)	$\displaystyle\int \frac{5}{x}+\frac{4}{(1-x)}+\frac{8}{(2x+1)}\,\mathrm{d}x = 5\ln\lvert x\rvert - 4\ln\lvert 1-x\rvert + 4\ln\lvert 2x+1\rvert \ (+c)$	M1A1ftA1
	$\left[5\ln\lvert x\rvert - 4\ln\lvert 1-x\rvert + 4\ln\lvert 2x+1\rvert\right]_2^4$ $=\left(5\ln 4 - 4\ln 3 + 4\ln 9\right) - (5\ln 2 - 4\ln 1 + 4\ln 5) = 5\ln 2 + 4\ln\left(\dfrac{3}{5}\right)$	dM1A1
		(5)
		(8 marks)

(a)

M1: Sets $5+17x-10x^2 = A(2x+1)(1-x)+Bx(2x+1)+Cx(1-x)$ condoning slips on the coefficients of $5+17x-10x^2$ only and finds values for *A*, *B* and *C*. The right-hand side must be correct. Do not be concerned how the values are formed, e.g. via substitution or simultaneous equations. Just look for a suitable identity followed by values for *A*, *B* and *C*.

A1: Two (out of three) correct values of $A=5,\ B=4,\ C=8$

A1: Accept either $A=5,\ B=4,\ C=8$ or $\dfrac{5}{x}+\dfrac{4}{(1-x)}+\dfrac{8}{(2x+1)}$ but condone $\dfrac{5}{x}-\dfrac{4}{(x-1)}+\dfrac{8}{(2x+1)}$ which is also correct (but not in the form requested).

(b)

M1: Attempts to integrate their $\dfrac{"5"}{x}+\dfrac{"4"}{(1-x)}+\dfrac{"8"}{(2x+1)}$.

Score for any one $\dfrac{"5"}{x} \to \alpha\ln\lvert x\rvert \quad \dfrac{"4"}{(1-x)} \to \beta\ln\lvert 1-x\rvert \quad \dfrac{"8"}{(2x+1)} \to \delta\ln\lvert 2x+1\rvert$

(Condone invisible modulus signs or invisible brackets if subsequent work implies them)

Condone $\beta\ln\lvert 1-x\rvert \to \beta\ln\lvert x-1\rvert$ **throughout (b)**

Be aware that if they incorrectly find *C*, e.g. $C=\dfrac{7}{2}$, then $\dfrac{7}{(4x+2)} \to \dfrac{7}{4}\ln(4x+2)$ is a correct follow through

A1ft: Two of $5\ln\lvert x\rvert - 4\ln\lvert 1-x\rvert + 4\ln\lvert 2x+1\rvert$ follow through their *A*, *B* and *C*.
Allow use of brackets instead of modulus signs or allow further work to imply them.

A1: $5\ln\lvert x\rvert - 4\ln\lvert 1-x\rvert + 4\ln\lvert 2x+1\rvert$ with or without the $+c$
Allow use of brackets instead of modulus signs or allow further work to imply them.

dM1: Substitutes in the limits, subtracts either way round and applies laws of logs correctly to achieve either an $p\ln 2$ term or a $q\ln\frac{3}{5}$ term.

It is dependent on the previous M and having integrated to a form $\alpha\ln|x|+\beta\ln|1-x|+\delta\ln|2x+1|$ or $\alpha\ln|x|+\beta\ln|x-1|+\delta\ln|2x+1|$

This is most likely going to be scored via the ln2 term.

The term in ln2 must be found from $\alpha\ln 4-\alpha\ln 2=\alpha\ln 2$ so the value of p and α must be the same

The term in $\ln\left(\frac{3}{5}\right)$ must be found from $\beta\ln 3+2\delta\ln 3-\delta\ln 5$

Condone calculations such as $\ln(-3)-\ln(-5)=\ln\frac{3}{5}$

A1: CSO: $5\ln 2+4\ln\left(\frac{3}{5}\right)$

It cannot be scored following incorrect integration. ISW after sight of this

Question Number	Scheme	Marks
5(a)	$\frac{dx}{dt}=\frac{2(1-t)-(-1)(3+2t)}{(1-t)^2}=\frac{5}{(1-t)^2}$ and $\frac{dy}{dt}=-2t$	
	At $t=2\Rightarrow\frac{dy}{dx}=\frac{(-1)^2}{5}\times\left(-2(2)\right)=-\frac{4}{5}$	M1A1
	$(-7,\ -3)$	B1
	$y+3=\frac{5}{4}(x+7)$	M1
	$5x-4y+23=0$	A1
		(5)
(b)	$x=\frac{3+2t}{1-t}\rightarrow t=\frac{x-3}{x+2}$	M1
	$y=1-\left(\frac{x-3}{x+2}\right)^2=\frac{(x+2)^2-(x-3)^2}{\left(x+2\right)^2}=\frac{10x-5}{\left(x+2\right)^2}$	dM1A1
	$x\neq-2$	B1
		(4)
		(9 marks)

FYI $x=\frac{3+2t}{1-t}=\frac{5}{1-t}-2$

(a) Note: A cartesian approach in (a) can score a maximum of 1 mark (the B1) as it does not satisfy the demand of the question.

M1: Attempts to find $\frac{dy}{dx}$ at $t=2$ by finding $\frac{dy}{dt}\div\frac{dx}{dt}$.

Condone slips when differentiating $\frac{dx}{dt}$ and $\frac{dy}{dt}$ but look for a minimum of

- $\frac{dx}{dt}=\frac{\alpha(1-t)\pm\beta(3+2t)}{(1-t)^2}$ with $\alpha>0$ or equivalent using the product rule or division and chain rule
- $\frac{dy}{dt}=\pm\varpi t$
- Substitution of $t=2$ into $\frac{dy}{dt}\div\frac{dx}{dt}$ which may have been simplified incorrectly. Note that a simplified $\frac{dy}{dx}=\frac{-2t\left(1-t\right)^2}{5}$

Note that using the quotient rule the wrong way around $\frac{uv'-vu'}{v^2}$ would be M0

A1: $-\frac{4}{5}$ following M1. It may be implied by a normal gradient of $\frac{5}{4}$

B1: $(-7,\ -3)$ which may be implied from terms within an attempt at the equation of the normal

M1: Attempts to find the equation of the normal at P. They must be using the negative reciprocal of their $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $t=2$ with their point P. It is dependent on having found $\frac{\mathrm{d}y}{\mathrm{d}x}$ from $\left(\text{their } \frac{\mathrm{d}y}{\mathrm{d}t}\right) \div \left(\text{their } \frac{\mathrm{d}x}{\mathrm{d}t}\right)$ Condone poor attempts at $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}y}{\mathrm{d}t}$ or what they believe are $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}y}{\mathrm{d}t}$ so 0, 0, 1, 1, 0 is possible. Condone one sign slip when using their coordinate of P. e.g $(-7, -3) \rightarrow (7, -3)$

If the form $y = mx + c$ is used the method must proceed as far as $c = \ldots$

A1: $5x - 4y + 23 = 0$ or any integer multiple with terms on one side of an equation. The order of the terms is not important. ISW after sight of a correct answer in a correct form

………

Other more complicated methods do exist so look carefully at what is attempted. For example, candidates could find $\frac{\mathrm{d}x}{\mathrm{d}y}$ at $t=2$ by finding $\frac{\mathrm{d}x}{\mathrm{d}t} \div \frac{\mathrm{d}y}{\mathrm{d}t}$ and use the value of $-\frac{\mathrm{d}x}{\mathrm{d}y}$ as the gradient of the normal.

………

(b)

M1: Attempts to rearrange the parametric equation for x to make t the subject. Expect to see cross multiplying (or equivalent) and collection of terms with $t = \frac{\mathrm{f}(x)}{\mathrm{g}(x)}$ where $\mathrm{f}(x)$ and $\mathrm{g}(x)$ are linear functions in x.

dM1: Substitutes their expression for t in terms for x into the parametric equation for y and attempts to manipulate the right-hand side into a single fraction. It is dependent on the previous method mark.

For example, look for $y = 1 - \left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right)^2 = \frac{(\mathrm{g}(x))^2 - (\mathrm{f}(x))^2}{(\mathrm{g}(x))^2}$

A1: $y = \frac{10x - 5}{(x+2)^2}$

B1: $x \neq -2$ or $k = -2$ but must follow correct work. E.g. Sight of $(x+2)$ o.e. on the denominator of either the expression for t or the expression for y.

………

Other more complicated methods do exist but are unlikely to lead to any success. If you see a solution that you feel deserves credit please send to review and alert your TL.

For example, if they start with $t = \sqrt{1-y}$ and then write down $x = \frac{3 + 2\sqrt{1-y}}{1 - \sqrt{1-y}}$ the first M mark will not be scored until they make $\sqrt{1-y}$ the subject (in terms of just x) which reverts to the original method

Question Number	Scheme	Marks
6(a)	Attempts $\pm\left((5\mathbf{i}+2\mathbf{j}+\mathbf{k})-(\mathbf{i}+2\mathbf{j}-3\mathbf{k})\right)=\pm(4\mathbf{i}+4\mathbf{k})$	M1
	e.g. $\mathbf{r}=\mathbf{i}+2\mathbf{j}-3\mathbf{k}+\lambda(4\mathbf{i}+4\mathbf{k})$ or $\mathbf{r}=5\mathbf{i}+2\mathbf{j}+\mathbf{k}+\lambda(4\mathbf{i}+4\mathbf{k})$ oe	A1
		(2)
(b)	$\overrightarrow{AC}=2\mathbf{i}+(\alpha-2)\mathbf{j}+8\mathbf{k}$	B1
	$\left\lvert\overrightarrow{AB}\right\rvert=\sqrt{4^2+4^2}$ or $\left\lvert\overrightarrow{AC}\right\rvert=\sqrt{2^2+(\alpha-2)^2+8^2}$	M1
	$\left\lvert\overrightarrow{AB}\right\rvert=\sqrt{32}$ and $\left\lvert\overrightarrow{AC}\right\rvert=\sqrt{68+(\alpha-2)^2}$	A1
	$\cos 45°=\dfrac{8+32}{\sqrt{32}\times\sqrt{68+(\alpha-2)^2}}$	dM1
	$\cos 45°=\dfrac{40}{\sqrt{32}\times\sqrt{68+(\alpha-2)^2}}\Rightarrow\alpha^2-4\alpha-28=0$	ddM1
	$\alpha=2\pm4\sqrt{2}$	A1
		(6)
		(8 marks)

Allow vectors to be written in any acceptable form. E.g. column vectors are fine

(a)

M1: Attempts to find the difference between the two vectors.
If no method is shown look for two correct components from $\pm(4\mathbf{i})$, $(0\mathbf{j})$, $\pm(4\mathbf{k})$

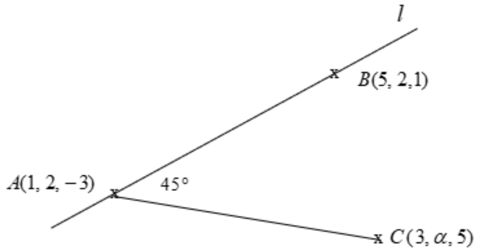
A1: Any correct equation for the line in the form given.
Examples of correct equations are;
$\mathbf{r}=\mathbf{i}+2\mathbf{j}-3\mathbf{k}+\lambda(4\mathbf{i}+4\mathbf{k})$, $\mathbf{r}=5\mathbf{i}+2\mathbf{j}+1\mathbf{k}+\mu(\mathbf{i}+\mathbf{k})$

$\mathbf{r}=\begin{pmatrix}5\\2\\1\end{pmatrix}+\lambda\begin{pmatrix}-4\\0\\-4\end{pmatrix}$, $\mathbf{r}=\begin{pmatrix}2\\2\\-2\end{pmatrix}+k\begin{pmatrix}2\\0\\2\end{pmatrix}$ and $\mathbf{r}=\begin{pmatrix}3\\2\\-1\end{pmatrix}+\delta\begin{pmatrix}-1\\0\\-1\end{pmatrix}$

Note that the LHS must be correct so $l=\mathbf{i}+2\mathbf{j}-3\mathbf{k}+\lambda(4\mathbf{i}+4\mathbf{k})$ is A0

Condone the vectors not being bold or underlined

Condone incorrect vector notation for the M but not the A mark. E.g. $\mathbf{r}=\begin{pmatrix}5\mathbf{i}\\2\mathbf{j}\\\mathbf{k}\end{pmatrix}+\lambda\begin{pmatrix}-4\mathbf{i}\\0\\-4\mathbf{k}\end{pmatrix}$ is M1A0



(b) There are many ways to attempt this question.

The following is via the scalar product of $\overrightarrow{AB}$ and $\overrightarrow{AC}$. Note that it could be attempted via the scalar product of $\pm(1\mathbf{i}+1\mathbf{k})$ and $\overrightarrow{AC}$ instead of $\overrightarrow{AB}$ and $\overrightarrow{AC}$. Attempts using an incorrect vector pair, e.g $\overrightarrow{AB}$ and $\overrightarrow{BC}$, can **only score a maximum of 1 mark (1st M1)**

Condone incorrect vector notation throughout (b). E.g. $\overrightarrow{AC}=(2\mathbf{i}, (\alpha-2)\mathbf{j}, 8\mathbf{k})$

B1: A correct $\overrightarrow{AC}=2\mathbf{i}+(\alpha-2)\mathbf{j}+8\mathbf{k}$ (or $\overrightarrow{CA}=-2\mathbf{i}-(\alpha-2)\mathbf{j}-8\mathbf{k}$)
It must be simplified but may be implied by further work.
Allow in coordinate form $(2, \alpha-2, 8)$ if used correctly

M1: Attempts to find the magnitude of either their $\overrightarrow{AB}$ or their $\overrightarrow{AC}$
For example, for an attempt at the magnitude of $\overrightarrow{AC}$

- there should be attempt to subtract each component of $\overrightarrow{OA}$ and $\overrightarrow{OC}$
- the three resulting values should then be squared and added
- the answer to this is then square rooted

If the method is not apparent then look for two correct components

A1: Correct magnitude of both $|\overrightarrow{AB}|=\sqrt{32}$ and $|\overrightarrow{AC}|=\sqrt{68+(\alpha-2)^2}$ $\left(=\sqrt{72-4\alpha+a^2}\right)$
They must be simplified (as above) but can be scored following sign slips on either vector.
You can imply this mark from further work

dM1: Attempts the scalar product of $\pm'\overrightarrow{AB}'$and $\pm'\overrightarrow{AC}'$

- $\pm\overrightarrow{AB}$ and $\pm\overrightarrow{AC}$ must be attempted via a correct method. If the method is not directly seen it may be implied by two correct components
- There must be a similar correct attempt at the magnitudes of $\pm\overrightarrow{AB}$ and $\pm\overrightarrow{AC}$
- All should be connected by the equation $\pm\overrightarrow{AB}\bullet\pm\overrightarrow{AC}=|\pm\overrightarrow{AB}||\pm\overrightarrow{AC}|\cos 45$

ddM1: Allowable method to find at least one value of α from a quadratic equation.
For example,

- $(\alpha-2)^2=32\Rightarrow\alpha=...$
- $\alpha^2-4\alpha-28=0\Rightarrow\alpha=...$

This can only be awarded if the equation has real roots.
It is dependent upon both previous M's

A1: $\alpha=2\pm4\sqrt{2}$ or exact equivalent

Note: Alternative method via cosine rule

B1: $\overrightarrow{AC}=2\mathbf{i}+(\alpha-2)\mathbf{j}+8\mathbf{k}$ (or $\overrightarrow{CA}=-2\mathbf{i}-(\alpha-2)\mathbf{j}-8\mathbf{k}$) May be implied by further work.

M1: Attempts to find the magnitude of either $\overrightarrow{AB}$ or $\overrightarrow{AC}$

A1: $|\overrightarrow{AB}|=\sqrt{32}$ and $|\overrightarrow{AC}|=\sqrt{68+(\alpha-2)^2}$ $\left(=\sqrt{72-4\alpha+a^2}\right)$

dM1: Attempts $|BC|^2=|AB|^2+|AC|^2-2|AB||AC|\cos 45$

FYI $\left(2^2+(\alpha-2)^2+4^2\right)=\left(2^2+(\alpha-2)^2+8^2\right)+\left(4^2+4^2\right)-2\sqrt{\left(2^2+(\alpha-2)^2+4^2\right)}\sqrt{\left(4^2+4^2\right)}\times\frac{\sqrt{2}}{2}$

ddM1: Rearranges and proceeds to at least one value for α. This can only be awarded if the equation is quadratic and has real roots

A1: $\alpha=2\pm4\sqrt{2}$ or exact equivalent

Question Number	Scheme	Marks
7(a)	$u=\tan x \rightarrow \dfrac{du}{dx}=\sec^2 x$	B1
	$u^2=\tan^2 x=\sec^2 x-1 \rightarrow \displaystyle\int \frac{u+u^3}{(5+u^2)^3} \times \frac{1}{1+u^2}\, du$	M1dM1
	$\displaystyle\int \frac{u}{(5+u^2)^3}\, du$ *	A1*
		(4)
(b)	$\displaystyle\int \frac{u}{(5+u^2)^3}\, du = -\frac{1}{4}(5+u^2)^{-2}+c = -\frac{1}{4}(5+\tan^2 x)^{-2}\,(+c)$	M1A1
		(2)
		(6 marks)

(a)

B1: $\dfrac{du}{dx}=\sec^2 x$ or exact equivalent seen or used. For example, $\dfrac{dx}{du}=\dfrac{1}{1+u^2}$ is correct

M1: Attempts to use $\pm 1 \pm \tan^2 x = \sec^2 x$ in a correct place at least once. Implied by use of $\sec^2 x = \pm 1 \pm u^2$

dM1: Fully substitutes $u=\tan x$ proceeding to an integral in terms of <u>u only.</u>
It is dependent upon the previous M

Look for $\left(\displaystyle\int \frac{\tan x+\tan^3 x}{(4+\sec^2 x)^3}\, dx\right) \rightarrow \displaystyle\int \frac{u+u^3}{(4\pm 1\pm u^2)^3} \times f(u)\, du$ seen on at least one line

Condone a lack of an integral sign. You may never see the integral in terms of *x*. (It is in the question)

A1*: Proceeds with no errors in the body of their proof to $\displaystyle\int \frac{u}{(5+u^2)^3}\, du$ including the d*u* .

The d*x*'s and d*u*'s must be present and correct for their stage of working throughout the proof.

(b)

M1: Attempts to integrate an expression of the form $\displaystyle\int \frac{u}{(k+u^2)^n}\, du \rightarrow A(k+u^2)^{-n+1}$ where *A* is a constant and $n \neq 1, n \in \mathbb{N}$

A1: $-\dfrac{1}{4}(5+\tan^2 x)^{-2}+c$ or equivalent e.g. $-\dfrac{1}{4}(4+\sec^2 x)^{-2}+c$. (condone omission of +*c*)
This must be in terms of *x* and you can ISW after sight of a correct answer.

Question Number	Scheme	Marks
8(a)	$\dfrac{1}{\sqrt{4+x}}=4^{-\frac{1}{2}}\left(1+\dfrac{x}{4}\right)^{-\frac{1}{2}}$	M1
	$\left(1+\dfrac{x}{4}\right)^{-\frac{1}{2}}=1+\left(-\dfrac{1}{2}\right)\left(\dfrac{x}{4}\right)+\dfrac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2!}\left(\dfrac{x}{4}\right)^2+\ldots$	M1
	$\dfrac{1}{2}-\dfrac{1}{16}x+\dfrac{3}{256}x^2+\ldots$	A1A1
		(4)
(b)	$\dfrac{1}{2}+\dfrac{1}{16}x+\dfrac{3}{256}x^2+\ldots$	B1ft
		(1)
(c)	$\left(\dfrac{1}{2}-\dfrac{1}{16}x+\dfrac{3}{256}x^2\right)\left(\dfrac{1}{2}+\dfrac{1}{16}x+\dfrac{3}{256}x^2\right)=\dfrac{1}{4}+\dfrac{1}{128}x^2$	M1A1
		(2)
(d)	$\dfrac{1}{\sqrt{5}}\times\dfrac{1}{\sqrt{3}}=\text{"}\dfrac{1}{4}\text{"}+\text{"}\dfrac{1}{128}\text{"}=\text{"}\dfrac{33}{128}\text{"}$	M1
	$\text{"}\dfrac{128}{33}\text{"}\times 3=\dfrac{128}{11}$ (see alt (d) for alternative answer)	dM1A1
		(3)
Alt(d)	$\dfrac{1}{\sqrt{5}}\times\dfrac{1}{\sqrt{3}}=\text{"}\dfrac{1}{4}\text{"}+\text{"}\dfrac{1}{128}\text{"}=\text{"}\dfrac{33}{128}\text{"}$	M1
	$\dfrac{1}{\sqrt{15}}\times\dfrac{\sqrt{15}}{\sqrt{15}}=\dfrac{\sqrt{15}}{15}\approx\text{"}\dfrac{33}{128}\text{"}\Rightarrow\sqrt{135}=3\sqrt{15}\approx 3\times 15\times\text{"}\dfrac{33}{128}\text{"}=\dfrac{1485}{128}$	dM1A1
		(10 marks)

(a) It is scored B, M, A, A on epen. We are scoring it M, M, A, A

M1: Takes out a factor of 4 proceeding to $4^{-\frac{1}{2}}\left(1+\frac{x}{4}\right)^{-\frac{1}{2}}$ o.e such as $\frac{1}{2}\times\left(1+\frac{x}{4}\right)^{-\frac{1}{2}}$

M1: Attempts the binomial expansion.
Look for the correct binomial coefficient for the **third term with the correct power of** x. So, award for $\dfrac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2!}(\ldots x)^2$ or $\underline{\underline{\frac{3}{8}}}\times\ldots\underline{\underline{x^2}}$ where … could even be 1. If there is no intermediate working it is implied by the correct coefficient of $\frac{3}{128}x^2$ in the expansion of $\left(1+\frac{x}{4}\right)^{-\frac{1}{2}}$ or $\frac{3}{256}x^2$ in the expansion of $(4+x)^{-\frac{1}{2}}$

A1: Two correct and simplified of $\frac{1}{2}-\frac{1}{16}x+\frac{3}{256}x^2+\ldots$ So 1,0,1, 0 is possible

A1: All three correct and simplified of $\frac{1}{2}-\frac{1}{16}x+\frac{3}{256}x^2+\ldots$

Accept as a list. Condone any extra terms even if incorrect

Direct expansion: $(4+x)^{-\frac{1}{2}} = 4^{-\frac{1}{2}} + \left(-\frac{1}{2}\right)\times 4^{-\frac{3}{2}}x + \frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2}\times 4^{-\frac{5}{2}}x^2 + ... = \frac{1}{2} - \frac{1}{16}x + \frac{3}{256}x^2 + ...$

M1: For a constant term of $4^{-\frac{1}{2}}$ or $\frac{1}{2}$

M1: For a correct third term which may be un-simplified $\frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2}\times 4^{-\frac{5}{2}}x^2$

It is implied if they get $\frac{3}{256}x^2$ for the term in x^2

A1: Two correct and simplified of $\frac{1}{2} - \frac{1}{16}x + \frac{3}{256}x^2 + ...$

A1: All three correct and simplified of $\frac{1}{2} - \frac{1}{16}x + \frac{3}{256}x^2 + ...$

Accept as a list. Condone any extra terms even if incorrect

(b)

B1ft: For a correct answer $\frac{1}{2} + \frac{1}{16}x + \frac{3}{256}x^2 + ...$ which can be scored following a restart.

Allow as a list. Condone any extra terms even if incorrect.

Or follow through on their (a) $\alpha - \beta x + \delta x^2 + ...$ where α, β, δ are positive to (b) $\alpha + \beta x + \delta x^2 + ...$

Answers must be seen in (b).

(c)

M1: Attempts to multiply out the correct $\left(\frac{1}{2} - \frac{1}{16}x + \frac{3}{256}x^2\right)\left(\frac{1}{2} + \frac{1}{16}x + \frac{3}{256}x^2\right)$ or follow through using their answers to parts (a) and (b) and attempts to collect terms. **It must be an attempt** on $(\alpha - \beta x + \delta x^2)\times(\alpha + \beta x + \delta x^2)$ where α, β, δ are positive leading to $\alpha^2 + \kappa x^2$. Condone extra terms where the index is greater than 2, such as a term in x^4

Alternatively, attempts the expansion $(16 - x^2)^{-\frac{1}{2}} = 16^{-\frac{1}{2}}\left(1 - \frac{1}{16}x^2\right)^{-\frac{1}{2}}$ leading to $\frac{1}{4} \pm \kappa x^2$

A1: $\frac{1}{4} + \frac{1}{128}x^2$ ONLY Any extra terms would be A0. Condone = for $\approx$

(d) This cannot be scored from made up values of *a* and *b*.

M1: Substitutes $x = 1$ into both sides of their part (c) to achieve $\frac{1}{\sqrt{5}} \times \frac{1}{\sqrt{3}} = "\frac{1}{4}" + "\frac{1}{128}"$

dM1: Uses the fact that $\sqrt{135} = 3\times\sqrt{15}$ and finds a rational approximation for $\sqrt{135}$

Method 1: $\frac{1}{\sqrt{5}} \times \frac{1}{\sqrt{3}} = "\frac{1}{4}" + "\frac{1}{128}" \Rightarrow \frac{\sqrt{15}}{15} = "\frac{1}{4}" + "\frac{1}{128}" \Rightarrow \sqrt{135} = \left("\frac{1}{4}" + "\frac{1}{128}"\right)\times 45$

Method 2: $\frac{1}{\sqrt{5}} \times \frac{1}{\sqrt{3}} = "\frac{1}{4}" + "\frac{1}{128}" \Rightarrow \frac{1}{\sqrt{15}} = "\frac{1}{4}" + "\frac{1}{128}" \Rightarrow \sqrt{135} = \frac{3}{\left("\frac{1}{4}" + "\frac{1}{128}"\right)}$

The method marks M1, dM1 may be implied, e.g. by the calculation $\left("\frac{1}{4}" + "\frac{1}{128}"\right)\times 45$ without sight of $\sqrt{135}$

A1: Either $\frac{128}{11}$ (or $11\frac{7}{11}$) or $\frac{1485}{128}$ (or $11\frac{77}{128}$) (or accept both if found)

Question Number	Scheme		Marks
9(a)	$(V)=\pi\int_0^4\left(\cos x+\frac{1}{5}e^x\right)^2 dx$		B1
	$\left(\cos x+\frac{1}{5}e^x\right)^2=\cos^2 x+\frac{2}{5}e^x\cos x+\frac{1}{25}e^{2x}$		M1
	$=\frac{\cos 2x+1}{2}+\frac{2}{5}e^x\cos x+\frac{1}{25}e^{2x}$		dM1
	$=\pi\int_0^4\left(\frac{1}{2}+\frac{\cos 2x}{2}+\frac{2}{5}e^x\cos x+\frac{1}{25}e^{2x}\right)dx$		A1
			(4)
(b)	**Way I**	**Way Two**	
	$\int e^x\cos x\,dx=\pm e^x\sin x\pm\int e^x\sin x\,dx$	$\int e^x\cos x\,dx=e^x\cos x\pm\int e^x\sin x\,dx$	M1
	$=e^x\sin x-\int e^x\sin x\,dx$	$=e^x\cos x+\int e^x\sin x\,dx$	A1
	$\int e^x\cos x\,dx=e^x\cos x\pm e^x\sin x-\int e^x\cos x\,dx$ $2\int e^x\cos x\,dx=e^x\cos x\pm e^x\sin x$		dM1
	$\int e^x\cos x\,dx=\frac{1}{2}e^x(\cos x+\sin x)+(c)$		A1
			(4)
(c)	$\pi\int_0^4\left(\frac{1}{2}+\frac{\cos 2x}{2}+\frac{2}{5}e^x\cos x+\frac{1}{25}e^{2x}\right)dx$ $=(\pi)\left[\frac{1}{2}x+\frac{\sin 2x}{4}+\frac{2}{5}\left(\frac{1}{2}e^x(\sin x+\cos x)\right)+\frac{1}{50}e^{2x}\right]_0^4$ $2\pi\left(2+\frac{\sin 8}{4}+\frac{2}{5}\left(\frac{1}{2}e^4(\sin(4)+\cos(4))\right)+\frac{1}{50}e^8-\frac{11}{50}\right)$ $=\text{awrt } 290\ (\text{cm}^3)$		M1A1ft dM1 A1
			(4)
			(12 marks)

(a)

B1: A correct expression for the volume including limits and dx E.g. $\pi\int_0^4\left(\cos x+\frac{1}{5}e^x\right)^2 dx$

The correct limits may be implied by subsequent work. It can be awarded from a correct final answer (as it is not a given answer). Don't be concerned by the non- appearance of a V. Condone it being termed 'Area'

M1: Attempts to multiply out $\left(\cos x+\frac{1}{5}e^x\right)^2$ leading to $\cos^2 x\pm...e^x\cos x\pm...e^{2x}$

dM1: Uses the identity $\cos 2x=\pm2\cos^2 x\pm1$. Dependent upon previous M

A1: Fully correct $\pi\int_0^4\left(\frac{1}{2}+\frac{\cos 2x}{2}+\frac{2}{5}e^x\cos x+\frac{1}{25}e^{2x}\right)dx$

For this final mark

- condone either a missing dx or limits (but not both) if the candidate has already lost the B mark for the same reason
- condone missing limits if they have already appeared within an expression for the volume anywhere in part (a)

(b) Condone lack of d*x*'s throughout part (b).
If you see other methods that you feel deserve credit, and you cannot apply the scheme, then please send to review.

M1: Attempts integration by parts. Be generous unless they explicitly state the rule is $\int v\,\mathrm{d}u = uv + \int u\,\mathrm{d}v$

Score for Way One $\int e^x \cos x\ \mathrm{d}x = \pm e^x \sin x \pm \int e^x \sin x\ \mathrm{d}x$

Or Way Two $\int e^x \cos x\ \mathrm{d}x = e^x \cos x \pm \int e^x \sin x\ \mathrm{d}x$

A1: Correct intermediate form

Score for Way One $\int e^x \cos x\ \mathrm{d}x = e^x \sin x - \int e^x \sin x\ \mathrm{d}x$

Or Way Two $\int e^x \cos x\ \mathrm{d}x = e^x \cos x + \int e^x \sin x\ \mathrm{d}x$

dM1: Attempts integration by parts a second time. It is dependent upon having scored the previous M.

Score for $\int e^x \cos x\ \mathrm{d}x = e^x \cos x \pm e^x \sin x - \int e^x \cos x\ \mathrm{d}x$

AND rearranges to $2\int e^x \cos x\ \mathrm{d}x = e^x \cos x \pm e^x \sin x$

A1: CSO $\int e^x \cos x\ \mathrm{d}x = \frac{1}{2}e^x(\cos x + \sin x) + c$ o.e. condone lack of $+c$ and the dx

Withold this mark if there is incorrect or unexplained working within the proof

(c)

M1: Attempts to integrate their (a) condoning transcription errors on coefficients achieving $\alpha x + \beta \sin 2x + \chi\left(\text{"}\frac{1}{2}e^x(\cos x + \sin x)\text{"}\right) + \delta e^{2x}$ where α, β, χ and δ are constants following through on their attempted integration of $e^x \cos x$

They must have an attempted integration for part (b), which may be incorrect, which is then used in (c).

A1ft: $(\pi)\left(\frac{1}{2}x + \frac{\sin 2x}{4} + \frac{2}{5}\left(\frac{1}{2}e^x(\cos x + \sin x)\right) + \frac{1}{50}e^{2x}\right)$

Condone the absence of π and follow through their coefficients A, B, C and D from (a). The integration from part (c) must be correct.

So, look for $\int_0^4 \left(A + B\cos 2x + Ce^x \cos x + De^{2x}\right)\mathrm{d}x \to \left(Ax + \frac{B}{2}\sin 2x + C\left(\frac{1}{2}e^x(\cos x + \sin x)\right) + \frac{D}{2}e^{2x}\right)$

dM1: This requires

- the limits 4 and 0 must be substituted into an expression of the form $ax + b\sin 2x + ce^x(\cos x \pm \sin x) + de^{2x}$ either way around and subtracted. This may be implied by a decimal answer which you may need to check.
- the answer to the above multiplied by 2π (An appreciation that the shape consists of two solids)

A1: awrt 290 cm^3 (condone lack of units).

If they have correct integration, correct limits followed by awrt 290, the last two marks can be awarded. Note that an answer of 460 is achieved when candidates are working in degrees and scores the method mark but not the accuracy mark.

Question Number	Scheme	Marks
10	Assume there is an angle x, $90° < x < 180°$, $\left\lvert \dfrac{\cos 2x}{\cos x - \sin x} \right\rvert \ldots 1$ $\dfrac{\cos 2x}{\cos x - \sin x} \equiv \dfrac{(\cos x + \sin x)\cancel{(\cos x - \sin x)}}{\cancel{\cos x - \sin x}}$ $\lvert \cos x + \sin x \rvert \ldots 1 \Rightarrow (\cos x + \sin x)^2 \ldots 1 \Rightarrow 1 + 2\sin x \cos x \ldots 1 \Rightarrow \sin 2x \ldots 0$ This is a contradiction, as sin α is negative for all angles $180° < \alpha < 360°$, hence true *	B1 M1 dM1 A1*
		(4 marks)

Condone strict inequalities for the first 3 marks in this question

B1: Sets up the argument.

It requires

- words like 'assume there is an angle x' or 'let there be an angle x', condoning 'for all angles x'
- the domain '$90° < x < 180°$'
- the contradiction $\left\lvert \dfrac{\cos 2x}{\cos x - \sin x} \right\rvert \ldots 1$ but condone $\left\lvert \dfrac{\cos 2x}{\cos x - \sin x} \right\rvert > 1$

M1: Uses the double angle formula for cos 2x and simplifies $\dfrac{\cos 2x}{\cos x - \sin x}$ to $(\cos x + \sin x)$

It may be scored when prematurely removing the modulus before proceeding to $(\cos x + \sin x) \ldots 1$

dM1: Squares $\lvert(\cos x + \sin x)\rvert \ldots 1$ and uses suitable identities in proceeding to $\sin 2x \ldots 0$, condoning $\sin 2x >> 0$

It must be in a form in which the contradiction can easily be proven

A1: Deduces contradiction from opening statement and concludes. May consider the signs of sin x and cos x in the given range and deduce that the product is negative leading to the contradiction of the original assumption. Withold this mark if the modulus signs are not correctly dealt with. This mark is withheld for using strict inequalities

There will be many alternative approaches so you will need to look at what they have done to see if there is any credit worthy work. If in doubt please use review.

Alternative using *R*-alpha method

B1 M1: As above

dM1: Proceeds to the form $\left\lvert \sqrt{2}\cos(x - 45°) \right\rvert \ldots 1$ with the modulus seen

Alternatively, to the form $\left\lvert \sqrt{2}\sin(x + 45°) \right\rvert \ldots 1$ with the modulus seen

A1*: Considers $x \in (90°, 180°) \Rightarrow (x - 45°) \in (45°, 135°)$

Deduces that $\lvert\cos(x - 45°)\rvert < \dfrac{1}{\sqrt{2}}$ which means that $\left\lvert \sqrt{2}\cos(x - 45°) \right\rvert < 1$ so meaning that the given statement is true.

Alternatively

Considers $x \in (90°, 180°) \Rightarrow (x+45°) \in (135°, 225°)$ and deduces that $\left|\sin(x+45°)\right| < \frac{1}{\sqrt{2}}$ which means that $\left|\sqrt{2}\sin(x+45°)\right| < 1$ so meaning that the given statement is true

Alternative that proceeds via an identity in $\cos 2x$ **and** $\sin 2x$

M1: Squares $\left|\frac{\cos 2x}{\cos x - \sin x}\right| \ldots 1$ and uses suitable identities leading to an inequality in $\sin 2x$ and $\cos 2x$

$\cos^2 2x \ldots (\cos x - \sin x)^2 \Rightarrow \cos^2 2x \ldots 1 - \sin 2x$

dM1: Then uses Pythagoras identity $\Rightarrow 1 - \sin^2 2x \ldots 1 - \sin 2x \Rightarrow \sin 2x \ldots \sin^2 2x$

A1*: As $90° < x < 180°$, $\Rightarrow 180° < 2x < 360°$ and $\sin 2x < 0 \Rightarrow \sin 2x \ldots 1$ which is impossible and therefore you have a contradiction and the original statement is true

Alternative using logic for the last two marks

dM1 Attempts to explain the contradiction by referencing the domain $90° < x < 180°$ and corresponding range of values of (say) sin x, cos x. Must involve the modulus in their explanation or have squared previously. Stating that "$0 < \sin x < 1$ and $-1 < \cos x < 0$ for $90° < x < 180°$", followed by " so $\left|\sin x + \cos x\right| < 1$" would be a minimal but acceptable argument for dM1.

Then the A1* is given for a completely rigorous explanation (plus conclusion)

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