Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | | | | | |

Candidate Number | | | | |

Pearson Edexcel International Advanced Level

Thursday 10 June 2025

Morning (Time: 1 hour 30 minutes)

Paper reference **WFM03/01**

Mathematics

International Advanced Subsidiary/ Advanced Level
Further Pure Mathematics F3

You must have:
Mathematical Formulae and Statistics Tables (Yellow), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions:

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information:

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice:

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

Pearson

1: (a) Use the definition of $\cosh x$ in terms of exponentials to show that

$$2\cosh 5x \cosh x \equiv \cosh 6x + \cosh 4x$$

(2)

(b) Hence determine the exact values of x for which

$$\cosh 6x + \cosh 4x = 8\cosh x$$

giving your answers in terms of natural logarithms in simplest form.

(4)

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Question 1 continued

(Total for Question 1 is 6 marks)

2: Determine

(i) $\displaystyle\int \frac{1}{\sqrt{4x^2 + 8x + 9}}\, \mathrm{d}x$

(3)

(ii) $\displaystyle\int \operatorname{arcosh} 3x \, \mathrm{d}x$

(4)

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P 7 6 3 9 3 A 0 4 3 2

Question 2 continued

(Total for Question 2 is 7 marks)

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3:

$$\mathbf{M} = \begin{pmatrix} 1 & -1 & 4 \\ 3 & a & b \\ a & 1 & b \end{pmatrix} \quad \text{where } a \text{ and } b \text{ are constants}$$

Given that $\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ is an eigenvector of **M**,

(a) determine the corresponding eigenvalue.

(2)

(b) Hence determine the value of *a* and the value of *b*.

(2)

(c) Determine

(i) the other eigenvalues of **M**,

(3)

(ii) eigenvectors which correspond to these eigenvalues.

(3)

Question 3 continued

Question 3 continued

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Question 3 continued

(Total for Question 3 is 10 marks)

4: $$y = \text{arsinh}\, x + \text{arsinh}\left(\frac{1}{x}\right) \qquad x > 0$$

(a) Show that

$$\frac{dy}{dx} = \frac{x - 1}{x\sqrt{1 + x^2}}$$

(2)

(b) Hence determine the exact value of y for which $\frac{dy}{dx} = 0$, giving your answer as a simplified natural logarithm.

(3)

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Question 4 continued

(Total for Question 4 is 5 marks)

5:

$$I_n = \int_1^6 x^n (3x-2)^{-\frac{1}{2}} \, dx \qquad n \geqslant 0$$

(a) Show that, for $n \geqslant 1$

$$(3+6n)I_n = 4nI_{n-1} + 8 \times 6^n - 2$$

(5)

(b) Use the reduction formula in part (a) to determine the exact value of

$$\int_1^6 x^3 (3x-2)^{-\frac{1}{2}} \, dx$$

(4)

Question 5 continued

Question 5 continued

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Question 5 continued

(Total for Question 5 is 9 marks)

6: $$\mathbf{A} = \begin{pmatrix} 1 & k & 2 \\ 5 & 3 & -2 \\ 6 & -1 & 4 \end{pmatrix} \quad \text{where } k \text{ is a constant}$$

(a) Determine the value of k for which $\mathbf{A}$ is singular.

(3)

Given that $\mathbf{A}$ is non-singular,

(b) determine $\mathbf{A}^{-1}$, giving your answer in simplest form in terms of k.

(4)

Question 6 continued

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Question 6 continued

Question 6 continued

(Total for Question 6 is 7 marks)

7: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

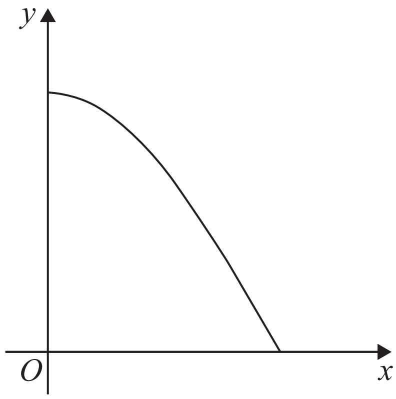

Figure 1

Figure 1 shows a sketch of the curve with equation

$$y = \cos 2x \qquad 0 \leqslant x \leqslant \frac{\pi}{4}$$

The curve is rotated through 2π radians about the x-axis.

(a) Show that the area of the curved surface generated is given by

$$S = 2\pi \int_0^{\frac{\pi}{4}} \cos 2x \sqrt{1 + 4\sin^2 2x}\, dx$$

(2)

(b) Hence, using the substitution $2\sin 2x = \sinh\theta$, show that

$$S = \frac{\pi}{4}\left(\ln\left(a + \sqrt{b}\right) + a\sqrt{b}\right)$$

where a and b are integers to be determined.

(7)

Question 7 continued

Question 7 continued

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Question 7 continued

(Total for Question 7 is 9 marks)

8: The plane Π_1 has equation

$$\mathbf{r} = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 7 \\ 5 \\ -6 \end{pmatrix} + \mu \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix}$$

where λ and μ are scalar parameters.

(a) Determine $(7\mathbf{i} + 5\mathbf{j} - 6\mathbf{k}) \times (-3\mathbf{i} + \mathbf{j} + 2\mathbf{k})$

(1)

(b) Hence show that the equation of Π_1 can be written in the form

$$\mathbf{r}.(8\mathbf{i} + 2\mathbf{j} + 11\mathbf{k}) = p$$

where p is a constant to be determined.

(2)

Given that

- the plane Π_2 has equation $x - y + z = 7$
- the planes Π_1 and Π_2 intersect in the line l_1

(c) determine an equation for l_1 giving your answer in the form $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = \mathbf{0}$ where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors.

(3)

Given also that

- the point A has coordinates (2, 1, 3)
- the point B has coordinates (3, 0, 2)
- the line l_2 passes through A and B

(d) determine the shortest distance between l_1 and l_2

(4)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 10 marks)

9: The hyperbola H has equation $\dfrac{x^2}{64} - \dfrac{y^2}{49} = 1$ and eccentricity e.

(a) Show that $e = \dfrac{\sqrt{113}}{8}$

(2)

The point $(8\sec\theta, 7\tan\theta)$, where $0 < \theta < \dfrac{\pi}{2}$, lies on H.

(b) Use calculus to show that the tangent to H at P has equation

$$\frac{x}{8}\sec\theta - \frac{y}{7}\tan\theta = 1$$

(3)

The tangent to H at P meets the y-axis at the point Q.

(c) Write down the coordinates of Q.

(1)

The normal to H at P

- has equation $8x\cos\theta + 7y\cot\theta = 113$
- meets the y-axis at the point R

(d) Write down the coordinates of R.

(1)

(e) Hence show that the circle with QR as a diameter passes through the foci of H.

(5)

Question 9 continued

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS