Mark Scheme

Summer 2023

Pearson Edexcel GCE

In Mathematics (9MA0)

Paper 01 Pure Mathematics

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 100.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- □ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.

   If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

General Principles for Pure Mathematics Marking

(*But note that specific mark schemes may sometimes override these general principles*)

<u>Method mark for solving 3 term quadratic:</u>

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \ldots$

2. Formula

Attempt to use the correct formula (with values for *a*, *b* and *c*)

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to $x = \ldots$

<u>Method marks for differentiation and integration:</u>

1. Differentiation

Power of at least one term decreased by 1. $(x^n \rightarrow x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n \rightarrow x^{n+1})$

<u>Use of a formula</u>

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

<u>Method mark</u> for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is <u>not</u> quoted, the method mark can be gained by implication from <u>correct</u> working with values but may be lost if there is any mistake in the working.

<u>Exact answers</u>

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question	Scheme	Marks	AOs
1	$x^{\frac{1}{2}}(2x-5) = ...x^{\frac{3}{2}} + ...x^{\frac{1}{2}}$ or $\frac{x^{\frac{1}{2}}(2x-5)}{3} = \frac{...x^{\frac{3}{2}} + ...x^{\frac{1}{2}}}{3}$	M1	1.1b
	$\frac{2x^{\frac{3}{2}}}{3} - \frac{5x^{\frac{1}{2}}}{3}$	A1	1.1b
	$\int \frac{2x^{\frac{3}{2}}}{3} - \frac{5x^{\frac{1}{2}}}{3}\,\mathrm{d}x = ...x^{\frac{5}{2}} \pm ...x^{\frac{3}{2}}\ \ (+c)$	dM1	1.1b
	$\frac{4}{15}x^{\frac{5}{2}} - \frac{10}{9}x^{\frac{3}{2}} + c$	A1	1.1b
		(4)	
		(4 marks)	

Notes

M1: Attempts to multiply out the brackets of the numerator and either writes the expression (or just the numerator) as a sum of terms with **indices**. Award for either one correct index of $...x^{\frac{3}{2}} + ...x^{\frac{1}{2}}$ which comes from a correct method. Condone appearing as terms on separate lines for this mark. The correct index may be implied later when e.g. $\sqrt{x} \to x^{\frac{3}{2}}$ or $\left(\sqrt{x}\right)^3 \to x^{\frac{5}{2}}$ after they have integrated.

The $\frac{1}{3}$ does not need to be considered for this mark.

A1: $\frac{2x^{\frac{3}{2}}}{3} - \frac{5x^{\frac{1}{2}}}{3}$ or equivalent e.g. $\frac{1}{3}\left(2x^{\frac{3}{2}} - 5x^{\frac{1}{2}}\right)$. Condone $\frac{2x^{\frac{3}{2}} - 5x^{\frac{1}{2}}}{3}$ May be implied by further work.

The correct index may be implied later when e.g. $\sqrt{x} \to x^{\frac{3}{2}}$ or $\left(\sqrt{x}\right)^3 \to x^{\frac{5}{2}}$ after they have integrated.

Ignore incorrect integration notation around the terms. Ignore any presence or absence of dx.

Be aware that a factor of $\frac{1}{3}$ may be taken outside of the integral so you may need to look at further work to award the first A mark if work on the two terms is done separately or in a list.

May be unsimplified and the two terms may appear in a list which is fine.

Coefficients must be exact.

dM1: Increases the power by one on an x^n term where n is a fraction. The index does not need to be processed.

e.g. $...x^{\frac{3}{2}+1}$ or $...x^{\frac{1}{2}+1}$ It is dependent on the previous method mark so at least one of the terms must have had a correct index.

Note that integrating the numerator and denominator e.g. $\frac{2x^{\frac{3}{2}}}{3} - \frac{5x^{\frac{1}{2}}}{3} \to \frac{...x^{\frac{5}{2}}}{3x} - \frac{...x^{\frac{3}{2}}}{3x}$ is dM0.

A1: $\frac{4}{15}x^{\frac{5}{2}}-\frac{10}{9}x^{\frac{3}{2}}+c$ and including the constant or simplified exact equivalent such as

$\frac{4}{15}\sqrt{x^5}-\frac{10}{9}\sqrt{x^3}+c$ or $\frac{4}{15}x^{2.5}-\frac{10}{9}x^{1.5}+c$ or $\frac{1}{45}\left(12x^{\frac{5}{2}}-50x^{\frac{3}{2}}\right)+c$ or $\frac{x^{\frac{3}{2}}}{45}(12x-50)+c$. Fractions must be in their lowest terms and indices processed.

Do not accept e.g. $0.267x^{\frac{5}{2}}-1.11x^{\frac{3}{2}}+c$ but allow $0.2\dot{6}x^{\frac{5}{2}}-1.\dot{1}x^{\frac{3}{2}}+c$

Isw once a correct answer is seen but withhold this mark if there is spurious notation around their final answer e.g. $\int \frac{4}{15}x^{\frac{5}{2}}-\frac{10}{9}x^{\frac{3}{2}}+c\ \mathrm{d}x$ is M1A1dM1A0

Alternative method using integration by parts example

M1: e.g. $\int x^{\frac{1}{2}}(2x-5)\ \mathrm{d}x = ...x^{\frac{3}{2}}(2x-5)-\int ...x^{\frac{3}{2}}\ \mathrm{d}x$ (applies integration by parts correctly to typically achieve this form – the $(2x-5)$ may also be split up as well – send to review if unsure how to mark)

This may also be done the other way round e.g. $\int x^{\frac{1}{2}}(2x-5)\ \mathrm{d}x = ...x^{\frac{1}{2}}(x^2-5x)-\int ...x^{\frac{3}{2}} \pm ...x^{\frac{1}{2}}\ \mathrm{d}x$

The $\frac{1}{3}$ does not need to be considered for this mark.

A1: A correct intermediate stage applying integration by parts with correct coefficients.

e.g. $\int \frac{x^{\frac{1}{2}}(2x-5)}{3}\ \mathrm{d}x = \frac{2}{3}x^{\frac{3}{2}}\left(\frac{2x-5}{3}\right)-\int \frac{4}{9}x^{\frac{3}{2}}\ \mathrm{d}x$ (or unsimplified equivalent).

Coefficients must be exact. (See main scheme notes above) The other way round this could appear as

e.g. $\int \frac{x^{\frac{1}{2}}(2x-5)}{3}\ \mathrm{d}x = \frac{1}{3}x^{\frac{1}{2}}(x^2-5x)-\frac{1}{6}\int x^{\frac{3}{2}}-5x^{\frac{1}{2}}\ \mathrm{d}x$. Condone a missing d$x$. May be implied.

dM1: Increases the power by one on an x^n term where n is a fraction e.g. $\int ...x^{\frac{3}{2}}\ \mathrm{d}x \to ...x^{\frac{5}{2}}$ The index does not need to be processed. It is dependent on the previous method mark.

A1: $\frac{4}{15}x^{\frac{5}{2}}-\frac{10}{9}x^{\frac{3}{2}}+c$ or exact simplified equivalent. (See main scheme notes above)

Alternative method using the substitution method

M1: e.g. let $u=x^{\frac{1}{2}} \Rightarrow \int ...u^4+...u^2\ \mathrm{d}u$ (uses a substitution to express the integral in terms of another variable. Allow slips with the coefficients, but the indices should be correct for their substitution)

The $\frac{1}{3}$ does not need to be considered for this mark.

A1: e.g. $\int \frac{4u^4}{3}-\frac{10u^2}{3}\ \mathrm{d}u$ or unsimplified equivalent. Coefficients must be exact. See main scheme notes above). May be implied by further work. Condone a missing dx.

dM1: $\int ...u^4+...u^2\ \mathrm{d}u \to ...u^5+...u^3$ (increases the power by one on at least one of their indices – does not need to be processed. It is dependent on the previous method mark.

A1: $\frac{4}{15}x^{\frac{5}{2}}-\frac{10}{9}x^{\frac{3}{2}}+c$ or exact simplified equivalent. (See main scheme notes above)

There may be alternative substitutions, but the same marking principles apply.

Question	Scheme	Marks	AOs
2(a)	$\left(\mathrm{f}(a)=\right)\ 4a^3+5a^2-10a+4a=0 \Rightarrow a(....)=0$	M1	3.1a
	$a(4a^2+5a-6)=0$ *	A1*	1.1b
		(2)	
(b)(i)	$a=\dfrac{3}{4}$	B1	2.2a
(ii)	$4x^3+5x^2-10x+4\times"\dfrac{3}{4}"=3 \Rightarrow 4x^3+5x^2-10x\ (=0)$	M1	1.1b
	$x=0$	B1	1.1b
	$x=\dfrac{-5\pm\sqrt{185}}{8}$	A1	1.1b
		(4)	
			(6 marks)

Notes

(a)

M1: Attempts $\mathrm{f}(a)=0$ leading to an equation in a only and attempts to take a factor of a out. Condone slips. The = 0 may appear on a later line which is fine, but must be seen at some point in their solution in part (a).

A1*: Achieves the given answer with no errors including brackets.
Minimum acceptable is $4a^3+5a^2-10a+4a=0 \Rightarrow a(4a^2+5a-6)=0$
If the = 0 is absent at the start of their solution, it must appear before achieving the given answer.
Do not allow attempts to find the value of a and substitute that into $\mathrm{f}(x)$

More difficult alternative methods may be seen:

Alt 1: You may see attempts via division / inspection

$$
\begin{array}{r|llll}
 & 4x^2 & +(5+4a)x & & +(-10+5a+4a^2) \\
\hline
x-a & 4x^3 & +5x^2 & -10x & +4a \\
 & 4x^3 & -4ax^2 & & \\
 & & (5+4a)x^2 & -10x & \\
 & & (5+4a)x^2 & -a(5+4a)x & \\
 & & & (-10+5a+4a^2)x & +4a \\
 & & & (-10+5a+4a^2)x & -a(-10+5a+4a^2) \\
 & & & & -6a+5a^2+4a^3
\end{array}
$$

Then sets remainder $-6a+5a^2+4a^3=0$

M1: For dividing the cubic by $(x-a)$ leading to a quadratic quotient in x and a cubic remainder in a which is then set = 0 and attempts to take a factor of a out.

A1*: Completely correct with $a(4a^2+5a-6)=0$

Alt 2: You may also see a grid or an attempt at factorisation via inspection

	$4x^2$	$+(5+4a)x$	$+(-10+5a+4a^2)$
x	$4x^3$	$+(5+4a)x^2$	$(-10+5a+4a^2)x$
$-a$	$-4ax^2$	$-a(5+4a)x$	$-a(-10+5a+4a^2)$

OR $4x^3+5x^2-10x+4a \equiv (x-a)(4x^2+px-4)$ which should be followed by equating the x terms and x^2 terms to form two equations which can be solved simultaneously.

$-10=-ap-4$ and $5=-4a+p \Rightarrow p=5+4a$

$\Rightarrow -10=-a(5+4a)-4 \Rightarrow 4a^2+5a-6=0 \Rightarrow a(4a^2+5a-6)=0$

The above are examples. There may be other correct attempts so look at what is done.

M1: For an attempt to set up two simultaneous equations by equating coefficients for x and equating coefficients for x^2. Condone slips.

A1*: $4a^2+5a-6=0 \Rightarrow a(4a^2+5a-6)=0$ Completely correct with no errors.

(b) Mark (i) and (ii) together

(i)

B1: Deduces that $a=\frac{3}{4}$ only. May be implied by their resultant cubic. If they do (b)(ii) multiple times using other roots for which $a \neq \frac{3}{4}$, then the solutions arising from using the other roots $a \neq \frac{3}{4}$ must be rejected

(ii)

M1: Attempts to substitute their $a=\frac{3}{4}$ (which must be positive) into $\mathrm{f}(x)$, sets their $\mathrm{f}(x)=3$ and collects terms on one side (= 0 may be implied). Condone arithmetical and sign slips. Condone if they repeat this step using their other root(s).

B1: $(x=)\ 0$

A1: $(x=)\ \frac{-5\pm\sqrt{185}}{8}$ (**and these values only**) or exact equivalent (ignore 0 for this mark). Withhold this mark if the fraction line was clearly not intended to be under both terms. This mark cannot be scored if they proceed directly to the roots from $4x^3+5x^2-10x$ without taking a factor of or dividing by x first to **see the quadratic factor**. Isw once the correct answers are seen if they proceed to provide rounded answers after.

e.g. $4x^3+5x^2-10x+4\times"\frac{3}{4}"=3 \Rightarrow 0, \frac{-5\pm\sqrt{185}}{8}$ is M0B1A0

e.g. $4x^3+5x^2-10x=0 \Rightarrow 0, \frac{-5\pm\sqrt{185}}{8}$ is M1B1A0

e.g. $4x^3+5x^2-10x=0 \Rightarrow 4x^2+5x-10=0 \Rightarrow \frac{-5\pm\sqrt{185}}{8}$ is M1B0A1

e.g. $4x^3+5x^2-10x=0 \Rightarrow x(4x^2+5x-10)=0 \Rightarrow 0, \frac{-5\pm\sqrt{185}}{8}$ is M1B1A1

Question	Scheme	Marks	AOs
3(a)	$\left(\lvert\overrightarrow{OA}\rvert=\right)\sqrt{5^2+3^2+2^2}\ =\sqrt{38}$ *	B1*	1.1b
		(1)	
(b)	$\lvert\overrightarrow{OB}\rvert=\sqrt{2^2+4^2+a^2}\ =\sqrt{20+a^2}$ so when $a=5$ $\lvert\overrightarrow{OB}\rvert=\sqrt{20+25}=\sqrt{45}$	M1	1.1b
	$=5$	A1cso	2.3
		(2)	
		(3 marks)	

Notes

(a)

B1*: Shows the magnitude of $\lvert\overrightarrow{OA}\rvert$ is $\sqrt{38}$. Must see $\sqrt{5^2+3^2+2^2}$ or e.g. $\sqrt{25+9+4}$. We need to see how the value 38 or $\sqrt{38}$ is formed using the three components. Withhold this mark for incorrect working such as $\lvert\overrightarrow{OA}\rvert=5^2+3^2+2^2=38\Rightarrow\lvert\overrightarrow{OA}\rvert=\sqrt{38}$ but do not penalise poor notation to denote vectors or the magnitude as long as the intention is clear as to what they are finding (OA instead of $\lvert\overrightarrow{OA}\rvert$ is fine). Do not penalise if their square root does not go fully over all three terms as long as the intention is clear. May find $\lvert\overrightarrow{AO}\rvert$ instead which is acceptable.

$\left(\lvert\overrightarrow{OA}\rvert^2=\right)25+9+4=38\Rightarrow\left(\lvert\overrightarrow{OA}\rvert=\right)\sqrt{38}$ scores B1 (we see how 38 is found)

$\left(\lvert\overrightarrow{OA}\rvert^2=\right)25+9+4\Rightarrow\sqrt{38}$ scores B1 (we see how 38 is found)

$25+9+4=\sqrt{38}$ scores B0 (they are not equal)

$\left(\lvert\overrightarrow{OA}\rvert^2=\right)38\Rightarrow\left(\lvert\overrightarrow{OA}\rvert=\right)\sqrt{38}$ scores B0 (no method seen to show how 38 is found)

(b)

M1: Attempts to find $\lvert\overrightarrow{OB}\rvert$ (or $\lvert\overrightarrow{OB}\rvert^2$) in terms of a and substitutes in a positive integer for a to find a value for $\lvert\overrightarrow{OB}\rvert$ (or $\lvert\overrightarrow{OB}\rvert^2$). e.g. $\lvert\overrightarrow{OB}\rvert=\sqrt{20+a^2}\Rightarrow$ when $a=2\Rightarrow\sqrt{24}$. Also accept e.g. $\lvert\overrightarrow{BO}\rvert$ (or $\lvert\overrightarrow{BO}\rvert^2$).

Alternatively sets up an equation or an inequality e.g. $\sqrt{20+a^2}>\sqrt{38}$ and proceeds to $a^2>...$ (or $a^2=...$). Condone sign slips in their rearrangement only.

Allow the use of =, < or > for this mark. $"20"+a^2...\ 38\Rightarrow a^2..."18"$

(may be implied by sight of $\sqrt{18}=4.24...$)

A1: 5 cso (answer on its own with no incorrect working seen scores M1A1). Withhold this mark if $\lvert\overrightarrow{OB}\rvert$ (or $\lvert\overrightarrow{OB}\rvert^2$) is incorrect (or $\lvert\overrightarrow{BO}\rvert$ or $\lvert\overrightarrow{BO}\rvert^2$). Do not be concerned with the notation as long as the intention is clear or implied as to what they are trying to calculate (the calculations must be correct)

Withhold this mark if at any point they set $\lvert\overrightarrow{OB}\rvert<\lvert\overrightarrow{OA}\rvert$ but accept an argument leading to an answer of 5 from either $\lvert\overrightarrow{OB}\rvert>\lvert\overrightarrow{OA}\rvert$ or $\lvert\overrightarrow{OB}\rvert=\lvert\overrightarrow{OA}\rvert$

Question	Scheme	Marks	AOs
4a	$2\alpha+\frac{1}{2}\left(1-\frac{\alpha^2}{2}\right)$	M1	1.2
	$2\alpha+\frac{1}{2}\left(1-\frac{\alpha^2}{2}\right)=0\Rightarrow 2\alpha+\frac{1}{2}-\frac{\alpha^2}{4}=0 \quad \Rightarrow \alpha=...$	dM1	1.1b
	$\alpha=-0.243$ (3dp) only	A1	2.3
		(3)	
b	$\mathrm{f}'(0)=\frac{1}{2}\cos 0\Rightarrow ...\Rightarrow y=...x+3$	M1	1.1b
	$y=\frac{1}{2}x+3$	A1	1.1b
		(2)	
			(5 marks)

Notes

(a) **Accept to be in terms of α or another variable e.g. x**

Note: -0.243 **with no working is 0 marks**

M1: Fully substitutes $\cos x=1-\frac{x^2}{2}$ into the derivative.

dM1: Attempts to multiply out to achieve a 3TQ (= 0) **and** attempts to find a value for α. Condone slips. Allow solving the quadratic via any method (usual rules apply).

If they use a calculator then you may need to check this.

A1: $(\alpha=)-0.243$ only cao Can only be scored provided a correct 3TQ is seen. If both roots found then the other one must be rejected (or a choice made of -0.243 e.g. underlining it or a tick)

Condone $x=-0.243$

(b)

M1: Attempts to find the gradient of the curve when $x=0$ and achieves an equation of the form $y=\text{"}\mathrm{f}'(0)\text{"}x+3$.

$x=0$ must be fully substituted in and a value must be found for the gradient. Do not allow this mark if they attempt to use a changed gradient e.g. the gradient of the normal.

Also allow attempts using the small angle approximation:

$\mathrm{f}'(x)\approx 2x+\frac{1}{2}\left(1-\frac{x^2}{2}\right)$ when $x=0$, $\mathrm{f}'(0)=\text{"}\frac{1}{2}\text{"}\Rightarrow y=\text{"}\mathrm{f}'(0)\text{"}x+3$

A1: $y=\frac{1}{2}x+3$ or equivalent in the form $y=mx+c$ isw Stating just the values $m=0.5,\ c=3$ without the correct equation is A0

Question	Scheme	Marks	AOs
5(a)	$h = 0.2$	B1	1.1b
	$\frac{1}{2}\times"0.2"\times\{a+13.5+2(16.8+b+20.2+18.7)\}=17.59$	M1	1.1b
	e.g. $\Rightarrow a+13.5+2b+111.4=175.9 \Rightarrow a+2b=51$*	A1*	2.1
		(3)	
(b)	$a+16.8+b+20.2+18.7+13.5=97.2 \Rightarrow a+b=28 \Rightarrow a=...$ (or $b=...$)	M1	3.1a
	$a=5$ or $b=23$	A1	1.1b
	$a=5$ and $b=23$	A1	1.1b
		(3)	
		(6 marks)	

Notes

(a)

B1: States or uses $h = 0.2$ o.e.

M1: Forms the equation $\frac{1}{2}\times"0.2"\times\{a+13.5+2(16.8+b+20.2+18.7)\}=17.59$ o.e. but condone copying slips. They may have added some of the y values together so as a minimum accept e.g. $"0.1"\times\{a+13.5+2(55.7+b)\}=17.59$

Condone invisible brackets as long as they are recovered or implied in further work before achieving the given answer. Condone the use of $\approx$ for this mark.

Allow this mark if they add the areas of individual trapezia e.g.

$\frac{\text{their } h}{2}\{a+16.8\}+\frac{\text{their } h}{2}\{16.8+b\}+\frac{\text{their } h}{2}\{b+20.2\}+\frac{\text{their } h}{2}\{20.2+18.7\}+\frac{\text{their } h}{2}\{18.7+13.5\}$

Condone copying slips but it must be a complete method using all the trapezia. h must be numerical but condone $h=1$

A1*: A rigorous argument leading to $a+2b=51$ from correct working and no errors seen including brackets, although do not penalise a missing trailing bracket at the end e.g.

$\frac{1}{2}\times"0.2"\times\{a+13.5+2(16.8+b+20.2+18.7)=17.59\Rightarrow...\Rightarrow a+2b=51$ could score B1M1A1but

$\frac{1}{2}\times"0.2"\times a+13.5+2(16.8+b+20.2+18.7)=17.59\Rightarrow...\Rightarrow a+2b=51$ could score max B1M1A0

provided later work implied correct brackets.

Both sets of brackets must be dealt with correctly before proceeding to the final answer such that

e.g. $...\Rightarrow a+2b+124.9=175.9\Rightarrow a+2b=51$ is M1A1*

$...\Rightarrow a+13.5+33.6+2b+40.4+37.4=175.9\Rightarrow a+2b=51$ is M1A1*

$...\Rightarrow 0.1a+1.35+3.36+0.2b+4.04+3.74=17.59\Rightarrow a+2b=51$ is M1A1*

Note that $a+2b\approx 51$ as the **final** answer is A0*

(b)

M1: Attempts to form the equation $a+16.8+b+20.2+18.7+13.5=97.2$, condoning copying errors, (may just be stated as e.g. $a+b=28$ o.e.) and attempts to solve their equation simultaneously with the given equation (or condone their equation from part (a)). Do not be too concerned with the process here as calculators may be used. Score if values for a or b are reached from a pair of simultaneous equations.

A1: for $a=5$ or $b=23$

A1: for both $a=5$ and $b=23$

Question	Scheme	Marks	AOs
6(a)	$\frac{1}{2}a$	B1	2.2a
		(1)	
(b)	$\log_2 x(x+8) \Rightarrow \log_2 x + \log_2 (x+8)$	M1	1.2
	$= a+b$	A1	2.2a
		(2)	
(c)	e.g. $8+\frac{64}{x}=\frac{8x+64}{x}$	B1	1.1b
	$\log_2 \frac{8}{x}(x+8) = 3-\log_2 x+\log_2 (x+8)$	M1	1.1b
	$3+b-a$	A1	2.2a
		(3)	
			(6 marks)

Notes

Condone omission of base 2 in all parts.

(a)

B1: $\frac{1}{2}a$ or $\frac{a}{2}$ or $0.5a$ isw

(b)

M1: Takes a factor of x out of the bracket to achieve $\log_2 x(x+8)$ and attempts to apply the addition law of logs, usually leading to $\log_2 x+\log_2 \left(x+8\right)$. Condone missing brackets or omission of base 2. May be implied by a correct answer. Allow this mark to be scored if they write $\log_2 x+\log_2 x+\log_2 8$ (an answer of $2a+3$ can score M1A0)

$\log_2 x \times \log_2 \left(x+8\right)$ on its own is M0 but allow the mark to be scored if they proceed to $a+b$

A1: $a+b$ or simplified equivalent (a correct answer with no incorrect log work seen scores M1A1) isw

Note $\log_2 x \times \log_2 \left(x+8\right) = a+b$ is M1A0 (allow the answer to imply the correct method but withhold the final mark)

(c)

B1: Writes $8+\frac{64}{x}$ as a single fraction e.g. $\frac{8x+64}{x}$ or $\frac{8}{x}(x+8)$ or $8x^{-1}(x+8)$ or $8\left(\frac{x^2+8x}{x^2}\right)$ which may be implied by later work e.g. $\log_2 8-\log_2 x+\log_2 \left(x+8\right)$

M1: Attempts to apply the laws of logs, uses $\log_2 8=3$ and proceeds to $3\pm\log_2 x\pm\log_2 \left(x+8\right)$ (or equivalent since $\pm\log_2 x$ may appear as $\pm\log_2 \frac{1}{x}$ or $\pm\log_2 x^{-1}$)

May be implied by $3\pm b\pm a$ and condone invisible brackets around $x+8$ and condone the omission of base 2.

Note that if they write $\log_2 \left(x+8\right)$ as $\log_2 x+\log_2 8$ this is M0

A1: $3+b-a$ or simplified equivalent (a correct answer with no incorrect log work seen is B1M1A1) isw

Note $\log_2 \frac{8}{x}(x+8) = 3 \div \log_2 x \times \log_2 (x+8) \Rightarrow 3-a+b$ is B1M1A0 (allow the answer to imply the correct method but withhold the final mark)

Note: You may see attempts to work backwards to the answer.

Question	Scheme	Marks	AOs
7(a)	$f(x) > 3$	B1	1.1b
		(1)	
(b)	$y = 3+\sqrt{x-2} \Rightarrow x = ...$	M1	1.1b
	$f^{-1}(x) = (x-3)^2 + 2$	A1	1.1b
	$x > 3$	B1ft	2.2a
		(3)	
(c)	$f(6) = 3+\sqrt{6-2} = 5 \Rightarrow g(\text{"}5\text{"}) = \dfrac{15}{\text{"}5\text{"}-3} = ...$	M1	1.1b
	$= \dfrac{15}{2}$	A1	1.1b
		(2)	
(d)	$3+\sqrt{a^2+2-2} = \dfrac{15}{a-3} \Rightarrow \text{"}a^2-9=15\text{"}$	M1	1.1b
	$a = 2\sqrt{6}$	A1	2.2a
		(2)	
			(8 marks)

Notes

(a)

B1: $f(x) > 3$ o.e.

e.g. $y > 3$, range > 3, $f(x) \in (3, \infty)$, $\{f(x) : f(x) > 3\}$, $f > 3$ **but not e.g.** $x > 3$, $f(x) \geqslant 3$, $[3, \infty)$

(b)

M1: Sets $y = 3+\sqrt{x-2}$ and attempts to make x the subject (or vice versa). Look for the correct order of operations so score for an expression of the form $(x =)\ (y \pm 3)^2 \pm 2$ or $(y =)\ (x \pm 3)^2 \pm 2$

A1: $f^{-1}(x) = (x-3)^2 + 2$ Also accept $f^{-1} : x \to (x-3)^2 + 2$. Condone $f^{-1} = (x-3)^2 + 2$ (or $f^{-1} = y = (x-3)^2 + 2$) but do not allow just $y = ...$ or $f^{-1} : y =$

Also accept other equivalent expressions such as $f^{-1}(x) = x^2 - 6x + 11$ (simplified or unsimplified)

B1ft: $x > 3$ or follow through on their part (a). The omission of $x \in \mathbb{R}$ is condoned.

Allow equivalent answers such as $x \in (\text{"}3\text{"}, \infty)$ or $\{x : x > \text{"}3\text{"}\}$

Note: It is also acceptable to define f^{-1} in any variable e.g. as $f^{-1}(t) = (t-3)^2 + 2$ $\quad t > 3$ as long as the variable is used consistently to score M1A1B1. If another variable is used other than x it must be fully defined e.g. $f^{-1}(t) = ...$ not just $f^{-1} = ...$

(c)

M1: Substitutes $x = 6$ into f and substitutes the result into g to find a value for $gf(6)$.

Allow an attempt to substitute $x = 6$ into $gf(x) = \dfrac{15}{\sqrt{x-2}}$ condoning slips. They must proceed to find a value. Condone arithmetical slips and bracket errors/omissions. Condone for M1 attempts where when dealing with $\sqrt{x-2}$ leads to two different answers e.g. $\dfrac{15}{\sqrt{6-2}} \to \pm\dfrac{15}{2}$

A1: $\dfrac{15}{2}$ only oe isw once a correct answer is seen

(d)

M1: Attempts to form the equation $3+\sqrt{a^2+2-2}=\dfrac{15}{a-3}$, and proceeds to a quadratic in *a* (usually $a^2=k$ or $a^2-k=15$ but condone arithmetical, miscopying and sign slips. Condone equations which would lead to complex roots.

May be implied by a correct exact answer.

Alternatively, they attempt to form the equation $a^2+2=\mathrm{f}^{-1}\mathrm{g}(a)\Rightarrow a^2+2=\left(\dfrac{15}{a-3}-3\right)^2+2$

$\Rightarrow (a+3)(a-3)=15\Rightarrow a^2-9=15$ (condone slips)

They should be square rooting both sides so that $\sqrt{a^2+2-2}\to a$, before multiplying both sides by $a-3$ and rearranging so that the a^2 term comes from their "$(a+3)(a-3)$"

May be implied by a correct exact answer for their quadratic in *a* but a correct decimal answer does not imply this mark.

A1: $(a=)\ 2\sqrt{6}$ or accept $\sqrt{24}$ (they must reject the negative solution if found as $\mathrm{f}\left(a^2+2\right)\neq \mathrm{g}(a)$ when $a=-2\sqrt{6}$) $\sqrt{6}\times\sqrt{4}$ is A0

isw $\sqrt{24}$ followed by $4\sqrt{6}$ (incorrect manipulation of the surd) but not followed by $\pm\sqrt{24}$ o.e.

A decimal answer on its own or multiple answers e.g. $\pm\sqrt{24}$ score A0.

Question	Scheme	Marks	AOs
8(a)	$OC \times 2.3 = 27.6$	M1	1.1b
	e.g. $OC = \dfrac{27.6}{2.3} = 12\,\text{m}$ *	A1*	2.1
		(2)	
(b)	e.g. $(2AOB =)\ \pi - 2.3$	M1	1.1b
	$\dfrac{\pi - 2.3}{2} \Rightarrow 0.421$ rad *	A1*	2.1
		(2)	
(c)	Area $OCDE = \dfrac{1}{2} \times 12^2 \times 2.3$	M1	1.1b
	$= 165.6\,(\text{m}^2)$ (accept awrt 166)	A1	1.1b
	$(OB =)\ \dfrac{35 - 27.6}{2} + 12 = 15.7\,\text{m}$	B1	2.1
	Area of OAB (or OFG) $= \dfrac{1}{2} \times "15.7" \times 7.5 \times \sin 0.421 \quad (= 24.0...\text{m}^2)$	M1	1.1b
	Total area = $"165.6" + 2 \times "24.1"$	dM1	3.1a
	$=$ awrt $214\,(\text{m}^2)$	A1	1.1b
		(6)	
			(10 marks)

Notes

(a)

M1: Uses $l = r\theta$ with $l = 27.6$ and $\theta = 2.3$ correctly substituted in (may be labelled differently in their equation). Values just need to be embedded in an equation or accept an expression for *OC* e.g. $\dfrac{27.6}{2.3}$.
May work in degrees which is acceptable.
Condone an alternative letter being used to denote *OC* such as *r*
Alternatively, they use $l = r\theta$ with $r = 12$ and $\theta = 2.3$ and verify that $l = 27.6\,\text{m}$

A1*: Achieves an expression for *OC* before proceeding to $OC = 12\,(\text{m})$ with no errors seen (condone lack of units)
They must show at least $\dfrac{27.6}{2.3} \Rightarrow OC = 12\,(\text{m})$ which can score M1A1*
$r = \dfrac{27.6}{2.3} = 12$ is M1A1* (condone alternative letters for *OC*)
BUT e.g. $\dfrac{27.6}{2.3} = 12(\text{m})$ on its own is M1A0*
e.g. $OC \times 2.3 = 27.6 \Rightarrow OC = 12(\text{m})$ is M1A0*

In the alternative method they verify $l = 27.6$ and conclude that $OC = 12\,\text{m}$
We must see the calculation $12 \times 2.3 = 27.6$ and conclude that $OC = 12\,(\text{m})$
e.g. arc $= 12 \times 2.3 = 27.6$ so $OC = 12(\text{m})$ is M1A1* whereas $12 \times 2.3 = 27.6$ is M1A0*
Also allow e.g. if $OC = 12(\text{m})$ then $12 \times 2.3 = 27.6$ ✓ is M1A1*
If they work in degrees and use rounded values this scores A0* (If they work with e.g. $\dfrac{414}{\pi}$ to keep the angle exact then A1* can still be scored)

(b)

M1: Attempts to subtract 2.3 from π (which may be implied by an expression for *AOB* which is not the given answer)

e.g. $\frac{1}{2}(\pi - 2.3)$ or $\frac{\pi}{2} - 1.15$ score M1

May work in degrees e.g. $180 - \text{awrt}132$ is M1

Condone invisible brackets e.g. $\pi - 2.3 \div 2$ can still score M1.

A1*: Achieves 0.421 (rad) with no errors seen (ignore any side working which is not part of their main solution). Look for a correct expression which is awrt 0.421 before proceeding to the answer. Alternatively, they may write

e.g $2AOB = \pi - 2.3\ (= 0.8415..) \Rightarrow AOB = 0.421$

Condone if they do not round their answer at the end to 0.421.

Condone lack of units. Condone poor labelling of other angles and it does not require *AOB* = to score this mark, but do not accept e.g. *ABO* =

If they work in degrees then withhold this mark if they do not show the conversion back to radians.

e.g. $\frac{\pi - 2.3}{2} = 0.421\text{(rad)}$ is M1A1*

e.g. $\frac{180 - \text{awrt}131.8}{2} \div \frac{180}{\pi} = 0.421$ (rad) is M1A1* (conversion from degrees to radians seen)

e.g. $\pi - 2.3 \div 2 = 0.421$ (rad) is M1A0* (invisible/lack of brackets)

e.g. $\pi - 2.3 = \frac{0.842...}{2} = 0.421$ M1A0* (incorrect joined statement)

(c)

M1: Attempts to use $A = \frac{1}{2}r^2\theta$ with $r = 12$ and $\theta = 2.3$ The values embedded in the formula is sufficient for this mark. May be implied by a correct answer or further work. Look out for alternative more complex ways to find the area of the sector. e.g. area of semicircle – area of two sectors with $r = 12$ and $\theta = 0.421$

A1: awrt 166 (may be implied by later work)

B1: A correct expression or value for the length *OB* or *OF* which may be a part of a calculation (may see 15.7 in the equation to find the area of *AOB*). May be implied by a correct value for the area of a congruent triangle (or both)

M1: Attempts to find the area of at least one of the two congruent triangles using their *OB* found from $\frac{35 - 27.6}{2} + 12\ (= 15.7)$, $OA = 7.5$ and $\theta = 0.421$ in $\frac{1}{2} \times OA \times OB \times \sin C$ (may work in degrees)

Be aware that omitting sine in the formula may give a value close to the area of the triangle which would be M0. **Condone use of $\theta = 0.4$ or $\theta = 0.42$** if they have rounded angle *AOB*.

The values embedded in the expression is sufficient to score the mark or may be implied by the value.

Look out for more complex methods to find the area of one or both of the two congruent triangles

e.g. they may split the congruent triangle into two right angled triangles and add the separate areas.

Other alternatives

e.g. finding the area of the trapezium *ABFG*:

$BF = 2\times 15.7\cos 0.421$

Area of $AOB = \frac{1}{2}\left(\left(\frac{15+2\times 15.7\cos 0.421}{2}\right)\times 15.7\sin 0.421 - \frac{1}{2}\times 15.7^2\times \sin 2.3\right)$ o.e.

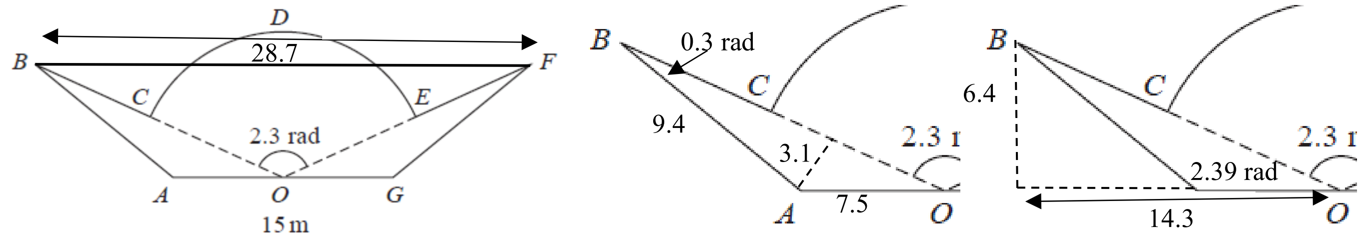
e.g. finding the length *AB* and either angle *OAB* or angle *ABO*:

$AB^2 = 15.7^2 + 7.5^2 - 2\times 15.7\times 7.5\times \cos 0.421 \Rightarrow AB = 9.37...$

$\frac{\sin ABO}{7.5} = \frac{\sin 0.421}{9.37...} \Rightarrow ABO = 0.333...$ or $\frac{\sin OAB}{15.7} = \frac{\sin 0.421}{9.37...} \Rightarrow OAB = 2.4$

Area of ABO $\frac{1}{2}\times 15.7\times 9.37...\times \sin 0.333...$ or $\frac{1}{2}\times 7.5\times 9.37...\times \sin 2.4$

Approximate values are shown below for some of the lengths you may see in calculations:



dM1: Solves the problem by combining appropriate areas together which result in the total area of the concert stage (usually the sum of the areas of the **two** congruent triangles and the area of the sector).
It is dependent on the previous method marks and the B mark.

A1: awrt 214 (m^2) (condone lack of units). Must follow from a correct method. Isw if they round incorrectly.

Question	Scheme	Marks	AOs
9(a)	$\dfrac{12-3k}{3k+4}=\dfrac{k+16}{12-3k}$	M1	3.1a
	$3k^2-62k+40=0$ *	A1*	1.1b
		(2)	
(b)(i)	$3k^2-62k+40=0 \Rightarrow k=...$	M1	1.1b
	States $k=20$ and gives a reason e.g. that this gives a values of *r* such that $\lvert r\rvert<1$	A1	3.2a
(ii)	$a=64$ **and** $r=-\dfrac{3}{4}$ (or allow $a=6$ **and** $r=\dfrac{5}{3}$)	B1	1.1b
	$S_\infty=\dfrac{"64"}{1-"\left(-\frac{3}{4}\right)"}=...$	M1	3.1a
	$S_\infty=\dfrac{256}{7}$	A1	1.1b
		(5)	
			(7 marks)

Notes

(a)

M1: Forms a correct equation linking the three terms. Condone invisible brackets if implied by further work.
Possible equations below (which are not exhaustive) should use *n*th term or sum of series formulae

e.g. $\dfrac{12-3k}{3k+4}=\dfrac{k+16}{12-3k}$ or $\left(\dfrac{12-3k}{3k+4}\right)^2=\dfrac{k+16}{3k+4}$ or $(12-3k)^2=(3k+4)(k+16)$ or

$(3k+4)\left(\dfrac{k+16}{12-3k}\right)=12-3k$ or $(12-3k)\left(\dfrac{12-3k}{k+16}\right)=3k+4$ or

$$3k+4+12-3k+k+16=\frac{(3k+4)\left(1-\left(\dfrac{k+16}{12-3k}\right)^3\right)}{1-\dfrac{k+16}{12-3k}}$$ (sum of three terms)

A1*: Achieves the given quadratic with no errors including invisible brackets. It cannot be for just proceeding in one step from the starting equation to the given answer and usually will involve attempting to multiply out brackets or dealing with any fractions.

(b)(i)

M1: Attempts to solve the given quadratic achieving at least one value for *k*. Usual rules apply for solving a quadratic and this may be achieved directly from a calculator. (May also be implied by $\dfrac{2}{3}$)

A1: 20 and gives correct reasoning (**if *r* is found anywhere in part (i) then it must be correct**):
e.g. 20 since $|r| < 1$. e.g. since $|r| = 0.75 < 1$

e.g. by listing **at least two consecutive** terms for $k = 20$ (must be correct) e.g. 64, -48 do not withhold this mark if they proceed to make a comment e.g. "the numbers are getting smaller" as we are condoning this to mean they are referring to the magnitude of the numbers

e.g. when $k = 20$, $r = -\frac{3}{4}$ o.e. which is between 1 and -1 (condone "it is smaller than 1").

Do not accept a reason on its own which is just simply stating that the sequence is converging or equivalent such as "spiralling".

Allow reasoning which excludes $k = \frac{2}{3}$ e.g. $r = \frac{5}{3}$ which is greater than 1.

(ii)

Work may be seen in part (i), but must be used in part (ii) to score.

B1: $a = 64$ **and** $r = -\frac{3}{4}$ o.e. (or allow $a = 6$ **and** $r = \frac{5}{3}$ o.e.) May be implied by later work or a correct calculation using these values to find S_∞

M1: A full attempt to find S_∞ by using their value of *k* to reach a value for *r* such that $|r| < 1$ and a value for *a*. Condone sign slips in their calculations of *a* and *r* only. You may need to check this by substituting in their value for *k* if no calculations are seen.

They must substitute these values in to $\frac{a}{1-r}$ correctly so e.g. $a = 64,\ r = -\frac{3}{4} \Rightarrow S_\infty = \frac{64}{1-\frac{3}{4}}$ is M0.

They cannot just substitute in their *k* as *r* in the formula.

Do not allow attempts to manually calculate the values of lots of terms for this mark as this would not lead to the answer. $\sum_{n=1}^{\infty} 64 \times \left(-\frac{3}{4}\right)^{n-1}$ on its own is M0.

A1: $\frac{256}{7}$ cao. (Do not allow 36.6 as this is not S_∞) isw after a correct exact answer is seen.

Question	Scheme	Marks	AOs
10(a)(i)	Centre $(-3k,\ k)$	B1	2.2a
(ii)	$(x+3k)^2-9k^2+(y-k)^2-k^2+7=0 \Rightarrow (x+3k)^2+(y-k)^2=...$	M1	1.1b
	Radius $\sqrt{10k^2-7}$	A1ft	2.2a
		(3)	
(b)	$x^2+(2x-1)^2+6kx-2k(2x-1)+7=0 \Rightarrow ...x^2+(pk+q)x+rk+s\,(=0)$	M1	1.1a
	$5x^2+(2k-4)x+2k+8\ \ (=0)$	A1	1.1b
	$(2k-4)^2-4\times5\times(2k+8)=0 \Rightarrow k=...$	dM1	2.1
	Critical values $= 7\pm\sqrt{85}$	A1	1.1b
	$k<\text{"}7-\sqrt{85}\text{"}$ or $k>\text{"}7+\sqrt{85}\text{"}$ o.e.	ddM1	3.1a
	$k<7-\sqrt{85}$ or $k>7+\sqrt{85}$ o.e.	A1	2.5
		(6)	
			(9 marks)

Notes

(a)(i)

B1: $(-3k,\ k)$ o.e. Accept without brackets. May be written as $x=-3k,\quad y=k$

(a)(ii)

M1: Attempts to find r^2 by completing the square and collects terms outside the brackets on the other side of the equation. $(x\pm3k)^2-...k^2+(y\pm k)^2-...k^2+7=0 \Rightarrow (x\pm3k)^2+(y\pm k)^2=\pm ak^2\pm b$
Alternatively, they may try to use general formulae such as
$x^2+y^2+2fx+2gy+c=0 \Rightarrow r^2=f^2+g^2-c$
May also be implied by an expression for *r*.

A1ft: $\sqrt{10k^2-7}$ Condone unsimplified equivalent expressions such as $\sqrt{9k^2+k^2-7}$ and do not allow if this is written with the equation of the circle as $(x+3k)^2+(y-k)^2=10k^2-7$. It must be extracted from this and explicitly written as $\sqrt{10k^2-7}$ o.e.
Do not penalise if their square root does not go fully over all three terms as long as the intention is clear.
Only follow through on a centre of the form $(\pm3k,\ \pm k)$ which will lead to a radius of $\sqrt{10k^2-7}$
Do not allow $\pm\sqrt{10k^2-7}$ and do not isw e.g. if they divide their radius by 2 (thinking they had found the diameter) then A0

(b)

M1: Substitutes $y=2x-1$ into the equation of the circle or their manipulated equation of the circle from (a) and attempts to collect terms proceeding to $.x^2+(pk+q)x+rk+s=0$ where *p*, *q*, *r* and *s* are all non zero.
Condone arithmetical slips and do not be too concerned by the mechanics of their rearrangement.
May be implied by $5x^2+(2k-4)x+2k+8\ (=0)$ or by their values for *a*, *b* and *c* in their discriminant. Do not be concerned with the use of <, > or =

A1: $5x^2+(2k-4)x+2k+8\ \ (=0)$ (which may be implied by their *a*, *b* and *c* in their discriminant) Do not be concerned with the use of <, > or =
Check carefully the signs of $2k-4$ since $4-2k$ will lead to the same answers and should score maximum M1A0dM1A0ddM1A0

dM1: Attempts to find b^2-4ac for their 3TQ and attempts to find at least one critical value. Do not be too concerned by the mechanics of their rearrangement.
If they find the root(s) directly from a calculator you will need to check this. (condone decimals which may be rounded or truncated)
It is dependent on the first method mark. Do not be concerned with the use of <, > or =

A1: $7 \pm \sqrt{85}$

ddM1: Attempts to find the outside region for their critical values. It is dependent on the previous two method marks. (Must have **two values** to be able to score this mark)
States e.g. $k < "7-\sqrt{85}"$, $k > "7+\sqrt{85}"$ (condone $k \geqslant "7+\sqrt{85}"$, $k \leqslant "7-\sqrt{85}"$).
Condone for this mark $x \leftrightarrow k$ and e.g. $"7+\sqrt{85}" \leqslant k \leqslant "7-\sqrt{85}"$. Allow any equivalent expression including set notation which includes both outside regions. Do not penalise poor notation to indicate the outside regions. Condone e.g. "and" o.e for this mark.

A1: $k < 7-\sqrt{85}$ or $k > 7+\sqrt{85}$ or any equivalent expression including set notation which includes **both** outside regions.
e.g. $k < 7-\sqrt{85}$, $k > 7+\sqrt{85}$ $\left(-\infty, 7-\sqrt{85}\right) \cup \left(7+\sqrt{85}, \infty\right)$
$\left\{k : k \in \mathbb{R}, k < 7-\sqrt{85}\right\} \cup \left\{k : k \in \mathbb{R}, k > 7+\sqrt{85}\right\}$.
Allow " ," "or", " $\cup$ " or a space between the answers (or on different line) but do not accept "and", " $\cap$ "
If a variable is used it must be in terms of *k*
Do not allow e.g. $"k < 7-\sqrt{85}$ and $k > 7+\sqrt{85}"$ or $\left[-\infty, 7-\sqrt{85}\right] \cup \left[7+\sqrt{85}, \infty\right]$
isw provided there is no contradiction with the correct answer.

Alternative method:
Using the formula for the perpendicular distance of a point from a line via a Further Maths method
Send to review if you are unsure how to mark these

M1: Substitutes the values of $2x-y-1=0$ and $(-3k,\ k)$ into $d = \left|\frac{2(-3k)+(-1)k+(-1)}{\sqrt{2^2+(-1)^2}}\right|$

Condone sign slips.

A1: $(d=)\left|\frac{2(-3k)+(-1)k+(-1)}{\sqrt{2^2+(-1)^2}}\right|$

dM1: Attempts to proceed from $\left|\frac{2(-3k)+(-1)k+(-1)}{\sqrt{2^2+(-1)^2}}\right| < \sqrt{"10k^2-7"}$ to form a 3TQ (typically $k^2-14k-36>0$) and attempts to find the critical values as above via any valid method. Do not be concerned with the use of <, > or =

A1ddM1A1: As above

Question	Scheme	Marks	AOs
11(a)	$\log_{10} V = 3 \Rightarrow V = 10^3$	M1	1.1b
	$(V =)$ £1000	A1	3.4
		(2)	
(b)	e.g. $(\log_{10} b =)\ \dfrac{2.79-3}{10-0} = -0.021$ **or** $\log_{10} V = 3-0.021t$ **or** $10^{2.79} = "1000"b^{10}$	M1	1.1b
	e.g. $b = 10^{-0.021}$ $(=0.952796....)$ **or** $V = 10^3 \times 10^{-0.021t}$ **or** $b = \sqrt[10]{"0.61659..."}$	M1	3.1b
	$V = 1000 \times 0.953^{\,t}$	A1ft	3.3
		(3)	
(c)	e.g. $V = 1000 \times "0.953"^{24}$ $(= £315)$ or e.g. $\log_{10} V = 3 - "0.021" \times 24 \Rightarrow V = ...(= £313)$	M1	3.4
	which is close (to £320) so it is a suitable model	A1	3.2b
		(2)	
			(7 marks)

Notes

(a)

M1: Sets $\log_{10} V = 3$ and attempts to find a value for *a* or an expression for *V* when $t=0$. Score for sight of 10^3 or implied by the correct answer.
There may be more complicated routes to finding the initial value. e.g. finding a complete equation such as $\left(\log_{10} V = 3 + \dfrac{2.79-3}{10}t \Rightarrow \log_{10} V = 3 \Rightarrow V =\right) 10^3$
This mark can also be scored for the equation $V = 10^3 \times 10^{"-0.021"t}$ or $V = 1000 \times (....)^t$ but not $V = 10^{3-"0.021"t}$ (the 10^3 has not been split up from $10^{"-0.021"t}$)

A1: £1000 cao (including units) do not accept £10^3

(b) Mark (b) and (c) together. Note work seen in (a) must be used in (b) to score

M1: Either

- finds the gradient between the two points. Score for the expression $\dfrac{2.79-3}{10-0}$ o.e. e.g. -0.021
  Do not condone sign slips for this mark. May be implied by later work such as sight of $10^{-0.021}$.
- finds the equation for $\log_{10} V$ in terms of *t* e.g. $\log_{10} V = 3 - "0.021"t$ which may be unsimplified.
- forms the equation $10^{2.79} (= 616.5...) = "1000"b^{10}$ o.e. such as $2.79 = 3 + 10\log b$

M1: Attempts to find the value or an expression for *b* using their gradient or their equation
Score for either:

- the expression $10^{"-0.021"}$ o.e such as $10^{\frac{2.79-3}{10-0}}$ or may be implied by a correct value using their gradient. You may need to check this on your calculator.
- correctly proceeding from $\log_{10} V = 3 - "0.021"t$ to $V = 10^{3-"0.021"t}$ and splitting this into $V = 10^3 \times 10^{-"0.021"t}$

- attempting to equate coefficients:

$$\log_{10} V = \log_{10} a + (\log_{10} b)t \Leftrightarrow \log_{10} V = 3 - "0.021"t \Rightarrow \log_{10} b = "-0.021" \Rightarrow b = 10^{"-0.021"}$$

- using their equation $10^{2.79} = "1000"b^{10}$ or $2.79 = "3" + 10\log b$ and proceeding to e.g. $b = \sqrt[10]{"0.61659..."}$ or $b = 10^{"-0.021"}$

A1ft: Complete correct equation, follow through on their "1000" so score for $V = "1000" \times \left(\text{awrt } 0.953\right)^t$ or accept $V = "10^3" \times \left(\text{awrt } 0.953\right)^t$. Just stating the values of *a* and *b* is A0ft, but if the equation is written in (c) before substituting in $t = 24$ then this mark can be awarded.

(c) Mark (b) and (c) together

M1: A full and valid attempt to:

- either substitute $t = 24$ into their model of the form $V = ab^t$ where *a* is positive and finds a value for V
- or substitutes $t = 24$ into their model of the form $\log_{10} V = p + qt$ where *p* is positive and finds a value for V (if they only proceed as far as $\log_{10} V$ they would also have to find the value of $\log_{10} 320$)
- or substitutes $V = 320$ into their $V = 1000 \times 0.953^t$ o.e. and finds a value for *t*

(to enable the candidate to compare real life data with that of the model.)
Do not be too concerned with the mechanics of the solution but they must be attempting to find two values which can be compared (e.g. usually 320 and a value for *V*, but they could proceed to find $\log_{10} 320$ and compare with $\log_{10} V = 2.496$ when $t = 24$, or a value for *t* to compare with $t = 24$)
In cases with no working you will need to check the calculation.

A1: Compares their awrt £313-£315 with £320 or their awrt $t = 23.5 - 23.7$ with $t = 24$ or $\log_{10} 320 = 2.505...$ with 2.496 and makes a valid conclusion with a reason.
For this mark you require:

- correct calculations (if using percentage error allow this to be rounded to compare awrt £313-£315 with £320 then it will be in the range (1.4, 2.4). For £314.94 this is $= \text{awrt}\, 1.6\%$)
- a reason such as "the values are close", "the values are similar", "the values are approximately equal". Allow use of "$\approx$". Allow the calculation of the % error as reason.
- a statement that it is a "good" or "accurate" model or similar wording.

Note: Condone as a minimum e.g. "£314.94 and £320 so good model" (we accept the two values being stated here as a comparison that they are similar)

Do not allow incorrect statements such as the model is incorrect as it does not give £320.
Do not allow just "the model gives an underestimate of the true value" (does not comment sufficiently on whether the model is reliable)
Do not allow comments suggesting that the model is not reliable.
Note using the full value for *b* leads to 313.3285724…

Question	Scheme	Marks	AOs
12	$\dfrac{\sin(x+h)-\sin x}{h}$	B1	2.1
	$\dfrac{\sin x\cos h+\cos x\sin h-\sin x}{h}$	M1	1.1b
		A1	1.1b
	(As $h\to 0$), $\sin x\left(\dfrac{\cos h-1}{h}\right)+\cos x\left(\dfrac{\sin h}{h}\right)\to 0\times\sin x+1\times\cos x$	dM1	2.1
	so $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\cos x$ *	A1*	2.5
			(5 marks)

Notes

Throughout the question allow the use of $h=\delta x$ if used consistently
There is no requirement to see "gradient of chord" written down.

B1: Gives the correct fraction such as $\dfrac{\sin(x+h)-\sin x}{x+h-x}$ or $\dfrac{\sin x-\sin(x+h)}{-h}$ or $\dfrac{\sin(x+h)-\sin(x-h)}{2h}$ or $\dfrac{\sin(x-h)-\sin x}{x-h-x}$. Condone invisible brackets. May be implied by $\dfrac{\sin x\cos h+\cos x\sin h-\sin x}{h}$

M1: Uses the compound angle formula for $\sin(x\pm h)$ to give $\sin x\cos h\pm\cos x\sin h$

A1: Achieves $\dfrac{\sin x\cos h+\cos x\sin h-\sin x}{h}$ or equivalent (may be implied by further work).
Allow invisible brackets to be recovered.

dM1: **It is dependent on both the B and the M marks being awarded.**
Complete attempt to apply the given limits to the gradient of their chord. They must isolate $\left(\dfrac{\cos h-1}{h}\right)$ and replace with 0 and isolate $\left(\dfrac{\sin h}{h}\right)$ and replace with 1.

e.g. $\sin x\left(\dfrac{\cos h-1}{h}\right)+\cos x\left(\dfrac{\sin h}{h}\right)=\sin x\times 0+\cos x\times 1$

Accept as a minimum $\sin x\left(\dfrac{\cos h-1}{h}\right)+\cos x\left(\dfrac{\sin h}{h}\right)=\cos x$ (implying the application of the limits)

If they do not fully show $\left(\dfrac{\cos h-1}{h}\right)$ and $\left(\dfrac{\sin h}{h}\right)$ being isolated but proceed from

e.g. $\dfrac{\sin x(\cos h-1)+\cos x\sin h}{h}$ to $0\times\sin x+\cos x$ (or e.g. $0+\cos x$) then this can be implied and score dM1

$\dfrac{\sin x(\cos h-1)+\cos x\sin h}{h}=\cos x$ is dM0

Condone if limit notation remains within their expression after the limits have been applied.
e.g. $\lim\limits_{h\to 0}(\sin x\times 0+\cos x\times 1)$

Alternatively, condone use of the small angle approximations such that

$\dfrac{\sin x\cos h+\cos x\sin h-\sin x}{h}\to\dfrac{-\dfrac{h^2}{2}\sin x+h\cos x}{h}=-\dfrac{h}{2}\sin x+\cos x$ and replaces $\dfrac{h}{2}$ with 0

A1*: Uses correct mathematical language of limiting arguments to show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \cos x$ with no errors seen. (cso)

We need to see $h \to 0$ **at some point in their solution and linking** $\frac{\mathrm{d}y}{\mathrm{d}x}$ **with** $\cos x$ e.g.

- $\frac{\mathrm{d}y}{\mathrm{d}x} = ... = \lim_{h\to 0}\left(\sin x\left(\frac{\cos h - 1}{h}\right) + \cos x\left(\frac{\sin h}{h}\right)\right) = \cos x$
- $\frac{\mathrm{d}y}{\mathrm{d}x} = ... = \lim_{h\to 0}\left(-\frac{h}{2}\sin x + \cos x\right) = 0 \times \sin x + \cos x = \cos x$ (using small angle approximations)
- $\frac{\mathrm{d}y}{\mathrm{d}x} = ... = \frac{\sin x(\cos h - 1) + \cos x \sin h}{h} = \sin x \times 0 + 1 \times \cos x = \cos x$ as $h \to 0$

Condone $\mathrm{f}'(x)$ or y' in place of $\frac{\mathrm{d}y}{\mathrm{d}x}$

Give final A0 for no evidence of limiting arguments**:**

e.g. when $h = 0$ $\quad \frac{\mathrm{d}y}{\mathrm{d}x} = ... = \sin x\left(\frac{\cos h - 1}{h}\right) + \cos x\left(\frac{\sin h}{h}\right) = \sin x \times 0 + \cos x \times 1 = \cos x$ is A0

Do not allow the final A1 for just stating $\frac{\sin h}{h} = 1$ and $\frac{\cos h - 1}{h} = 0$ and attempting to apply these (without seeing e.g. $h \to 0$ at some point in their solution)

If they work in another variable (e.g. θ) then withhold the final mark. If they have mixed variables within some of their statements, then allow recovery but withhold the final mark.

Withhold this mark if there has been incorrect bracketing or invisible brackets when isolating $\sin x(\cos h - 1)$ e.g. $\frac{\sin x \cos h - 1 + \cos x \sin h}{h}$ but accept terms written as e.g. $\sin x \frac{\cos h - 1}{h}$ which do not require brackets. Condone a missing trailing bracket if the intention is clear.

Question	Scheme	Marks	AOs
13(a)	$a = 60$	B1	3.1b
	$2 = "60" - b(-20)^2 \Rightarrow b = ...$	M1	3.4
	$H = 60 - 0.145(t-20)^2$	A1	3.3
		(3)	
(b)	Height = 2 m	B1	3.4
		(1)	
(c)	$\alpha = 180$ or $\beta = 31$	M1	3.4
	$H = 29\cos(9t+180)° + 31$	A1	3.3
		(2)	
(d)	e.g. "The model allows for more than one circuit"	B1	3.5a
		(1)	
			(7 marks)

Notes

(a)

B1: $a = 60$ (may be seen in their final equation of the model or implied by 60 substituted for *a* in the model)

M1: Attempts to find *b* by substituting in $t = 0$, $H = 2$ and their *a* and proceeding to a value for *b*. May be seen as two simultaneous equations formed:
$2 = a - b(-20)^2$ and $60 = a - b(20-20)^2$ proceeding to a value for *b*

A1: $H = 60 - 0.145(t-20)^2$ or equivalent such as $H = -\frac{29}{200}t^2 + 5.8t + 2$ or $H = 60 - \frac{29}{200}(t-20)^2$ isw once a correct equation for the model is seen. Must be in terms of *H* and *t*. If they just state $a = 60,\ b = 0.145$ then A0
A correct answer with no working seen scores full marks.

(b)

B1: 2 cao (condone lack of units) This can be scored even if their model in (a) is incorrect (they may have used symmetry to determine this value)

(c)

M1: $(\alpha =)\ 180$ or $(\beta =)\ 31$ Condone $(\alpha =)\ \pi$

A1: $H = 29\cos(9t+180)° + 31$ or equivalent e.g. $H = -29\cos(9t) + 31$ isw once a correct equation for the model is seen. Must be in terms of *H* and *t*. If they just state $\alpha = 180,\ \beta = 31$ then A0.
A correct equation with no working seen scores both marks. Does not require the degree symbol.

(d)

B1: Score for a reason which makes reference to any of
- the alternative model allows repetition (allow phrases e.g. "multiple cycles", "repeated circuits", "cyclical", "periodic", "loops around", "the original model can only go up and down once")
- the alternative model after 2 minutes the carriage will be back at the start (e.g."at 2 mins, $H = 2$")
- the original/quadratic model after 40 seconds (or any time after this) will be negative (e.g. "the height will be negative which cannot happen")
- the original model after 2 minutes would not be back at the start

Do not allow vague responses on their own e.g. "the original model is a parabola"
If calculations are used then they must be correct using a correct model (allow rounded or truncated)
Look for a valid reason and ignore reference to anything else as long as it does not contradict

t	0	5	10	15	20	25	30	35	40	45	50	55	60	80	100	120
h	2	27	46	56	60	56	46	27	2	-31	-71	-118	-172	-462	-868	-1390

Question	Scheme	Marks	AOs
14	When n is even: $(2k+1)^3-(2k)^3=8k^3+12k^2+6k+1-8k^3=6k(2k+1)+1$ $\Rightarrow$ which is odd **or**	M1	3.1a
	When n is odd: $(2k+2)^3-(2k+1)^3=8(k^3+3k^2+3k+1)-(8k^3+12k^2+6k+1)=6k(2k+3)+7$ $\Rightarrow$ which is odd	A1	2.2a
	When n is even: $(2k+1)^3-(2k)^3=8k^3+12k^2+6k+1-8k^3=6k(2k+1)+1$ $\Rightarrow$ which is odd **and** When n is odd: $(2k+2)^3-(2k+1)^3=8(k^3+3k^2+3k+1)-(8k^3+12k^2+6k+1)=6k(2k+3)+7$ $\Rightarrow$ which is odd	dM1	2.1
	Hence odd for all $n\left(\in\mathbb{N}\right)$ *	A1*	2.4
		(4 marks)	

Notes

General guidance

It is likely that you will see a mixture of methods and approaches within some solutions.
Mark the approach which scores the highest number of marks.

There should be no errors in the algebra but allow e.g. invisible brackets to be "recovered".
Withhold the final mark if n is used instead of k or reference to $n\in\mathbb{R}$ but $n\in\mathbb{Z}^+$ is acceptable.

Main scheme algebraic method using e.g. $n=2k$ **and** $n=2k\pm1$
You will need to look at both cases and mark the one which is fully correct first.
Allow a different variable to k and may be different letters for odd and even

M1: For the key step attempting to find $(n+1)^3-n^3$ when $n=2k$ **or** $n=2k\pm1$ and attempting to multiply out and simplify to achieve a three term quadratic (allow equivalent representation of odd or even e.g. $n=2k+2$ **or** $2n\pm5$)
Condone arithmetical slips and condone the use of e.g. $n=2n$ and $n=2n\pm1$

A1: Complete argument for $n=2k$ **or** $n=2k+1$ (or e.g. $n=2k-1$) showing the result is odd.
Requires:

- Correct simplified quadratic expression e.g. $12k^2+6k+1$ (when $n=2k$), $12k^2+18k+7$ (when $n=2k+1$), $12k^2-6k+1$ (when $n=2k-1$) (may be factorised)
- A reason why the expression is odd e.g. $2k(6k+3)+1$ or may use a divisibility argument e.g. $\dfrac{12k^2+6k+1}{2}=6k^2+3k+\dfrac{1}{2}$
- Concludes "odd" o.e. (may be within their final conclusion)
There should be no errors in the algebra but allow e.g. invisible brackets if they are "recovered"
Condone the use of e.g. $n=2n$ and $n=2n\pm1$

dM1: Attempts to find $(n+1)^3-n^3$ when $n=2k$ **and** $n=2k\pm1$ and attempts to multiply out and simplify to achieve a three term quadratic (allow equivalent representation of odd or even e.g. $n=2k+2$, $2n\pm5$
Condone arithmetical slips and condone the use of e.g. $n=2n$ and $n=2n\pm1$

A1*: Complete argument for **both** $n=2k$ **and** $n=2k+1$ (or e.g. $n=2k-1$) showing the result is odd for all $n(\in\mathbb{N})$

Requires for both cases:

- Correct simplified expressions for both odd and even (which may be factorised)
- A reason why both of the expressions are odd
- Minimal conclusion (may be within their final conclusion)

An overall conclusion is also required. "Hence odd for all $n(\in\mathbb{N})$" Accept "hence proven", "statement proved", "QED"

The conclusion for when $n=2k$ and $n=2k+1$ may be within the final conclusion rather than separate which is acceptable e.g. "when $n=2k$ and when $n=2k+1$ the expression is odd, hence proven" (following correct simplified expressions and reasons)

	$(n+1)^3$	n^3	$(n+1)^3-n^3$
$n=2k-1$	$8k^3$	$8k^3-12k^2+6k-1$	$12k^2-6k+1$
$n=2k$	$8k^3+12k^2+6k+1$	$8k^3$	$12k^2+6k+1$
$n=2k+1$	$8k^3+24k^2+24k+8$	$8k^3+12k^2+6k+1$	$12k^2+18k+7$

Alternative methods:

Algebraic with logic example

M1: Attempts to multiply out the brackets and simplifies to achieve a three term quadratic. Condone arithmetical slips.

A1: Correct quadratic expression $3n^2+3n+1$

dM1: Attempts to factorise their quadratic such that $n^2+n\rightarrow n(n+1)$ within their expression e.g. $3n(n+1)+1$

A1*: Explains that e.g. $n(n+1)$ is always even as it is the product of two consecutive numbers so $3n(n+1)$ is odd $\times$ even $=$ even so $3n(n+1)+1$ is odd hence odd for all $n(\in\mathbb{N})$

Proof by contradiction example

M1: Attempts to multiply out the brackets and simplifies to achieve a three term quadratic.

A1: Correct quadratic expression $3n^2+3n+1$

dM1: Sets $3n^2+3n+1=2k$ (for some integer k) $\Rightarrow 3n(n+1)=2k-1$

A1*: Explains that $n(n+1)$ is always even as it is the product of two consecutive numbers so $3n(n+1)$ is odd $\times$ even $=$ even but $2k-1$ is odd hence we have a contradiction so $(n+1)^3-n^3$ is odd (for all $n(\in\mathbb{N})$. There must have been a correct opening statement setting up the contradiction e.g. "assume that there exists a value for n for which $(n+1)^3-n^3$ is even"

Solutions via just logic (no algebraic manipulation)
e.g.

If n is odd, then $(n+1)^3 - n^3$ is $\text{even}^3 - \text{odd}^3 = \text{even} - \text{odd} = \text{odd}$

If n is even, then $(n+1)^3 - n^3$ is $\text{odd}^3 - \text{even}^3 = \text{odd} - \text{even} = \text{odd}$

Both cases must be considered to score any marks and scores SC 1010 if fully correct

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Further Maths method (proof by induction

M1: Assumes true for $n = k$, substitutes $n = k+1$ into $(n+1)^3 - n^3$, multiplies out the brackets and attempts to simplify to a three term quadratic e.g. $3k^2 + 9k + 7$ Condone arithmetical slips

A1: $\left(\mathrm{f}(k+1) = 3k^2 + 3k + 1 + 6(k+1) =\right)$ $(k+1)^3 - k^3 + 6(k+1) = \mathrm{f}(k) + 6(k+1)$
which is odd + even = odd

dM1: Attempts to substitute $n = 1 \Rightarrow$ $(1+1)^3 - 1^3 = 7$ (which is true) (Condone arithmetical slips evaluating)

A1*: Explains that

- it is true when $n = 1$
- if it is true for $n = k$ then it is true for $n = k+1$
- therefore it is true for all $n\left(\in \mathbb{N}\right)$

Question	Scheme	Marks	AOs
15(a)	$...xe^x + ...e^x$	M1	1.1b
	$k\left(xe^x + e^x\right)$	A1	1.1b
	$\frac{d}{dx}\left(\sqrt{e^{3x}-2}\right) = \frac{1}{2}\times 3e^{3x}\left(e^{3x}-2\right)^{-\frac{1}{2}}$	B1	1.1b
	$\left(f'(x) =\right)\ \dfrac{(e^{3x}-2)^{\frac{1}{2}}("7"xe^x + "7"e^x) - "\frac{3}{2}"e^{3x}(e^{3x}-2)^{-\frac{1}{2}} \times "7"xe^x}{e^{3x}-2}$	dM1	2.1
	$f'(x) = \dfrac{7e^x\left(e^{3x}(2-x)-4x-4\right)}{2(e^{3x}-2)^{\frac{3}{2}}}$	A1	1.1b
		(5)	
(b)	$e^{3x}(2-x)-4x-4=0 \Rightarrow x(...e^{3x} \pm ...) = ...e^{3x} \pm ...$	M1	1.1b
	$\Rightarrow x = \dfrac{2e^{3x}-4}{e^{3x}+4}$ *	A1*	2.1
		(2)	
(c)	Draws a vertical line $x=1$ up to the curve then across to the line $y=x$ then up to the curve finishing at the root (need to see a minimum of 2 vertical and horizontal lines tending to the root)	B1	2.1
		(1)	
(d)(i)	$x_2 = \dfrac{2e^3-4}{e^3+4} = 1.5017756...$	M1	1.1b
	$x_2 =$ awrt 1.502	A1	1.1b
(ii)	$\beta = 1.968$	dB1	2.2b
		(3)	
(e)	$h(x) = \dfrac{2e^{3x}-4}{e^{3x}+4} - x$ $h(0.4315) = -0.000297...$ $\quad h(0.4325) = 0.000947...$	M1	3.1a
	Both calculations correct and e.g. states: • There is a change of sign • e.g $f'(x)$ is continuous • $\alpha = 0.432$ (to 3dp)	A1cao	2.4
		(2)	
			(13 marks)

Notes

(a)

M1: Attempts the product rule on xe^x (or may be $7xe^x$) achieving an expression of the form $...xe^x \pm ...e^x$. If it is clear that the quotient rule has been applied instead which may be quoted then M0.

A1: $k\left(xe^x + e^x\right)$ (e.g. $7(xe^x + e^x)$) or equivalent which may be unsimplified (may be implied by further work)

B1: $\left(\frac{d}{dx}\left(\sqrt{e^{3x}-2}\right) =\right) \frac{1}{2}\times 3e^{3x}\left(e^{3x}-2\right)^{-\frac{1}{2}}$ (simplified or unsimplified)

dM1: Attempts to use the quotient rule. It is dependent on the previous method mark.
Score for achieving an expression of the form

$$\left(\mathrm{f}'(x)=\right)\ \frac{(\mathrm{e}^{3x}-2)^{\frac{1}{2}}("7"x\mathrm{e}^{x}+"7"\mathrm{e}^{x})-"\dfrac{3}{2}"\mathrm{e}^{3x}(\mathrm{e}^{3x}-2)^{-\frac{1}{2}}\times"7"x\mathrm{e}^{x}}{\mathrm{e}^{3x}-2}$$ or equivalent (do not be concerned by the constants for their "7" or their "$\frac{3}{2}$" which may be both 1)

If it is clear that the quotient rule has been applied the wrong way round then score M0.
Alternatively, applies the product rule. Score for achieving an expression of the form

$$\left(\mathrm{f}'(x)=\right)\ (\mathrm{e}^{3x}-2)^{-\frac{1}{2}}("7"x\mathrm{e}^{x}+"7"\mathrm{e}^{x})-"\frac{3}{2}"\mathrm{e}^{3x}(\mathrm{e}^{3x}-2)^{-\frac{3}{2}}\times"7"x\mathrm{e}^{x}$$ or equivalent (do not be concerned by the constants for their "7" or their "$\frac{3}{2}$" which may be both 1)

Do not condone invisible brackets.

A1: $\left(\mathrm{f}'(x)=\right)\ \dfrac{7\mathrm{e}^{x}\left(\mathrm{e}^{3x}(2-x)-4x-4\right)}{2(\mathrm{e}^{3x}-2)^{\frac{3}{2}}}$ following a fully correct differentiated expression.

You may need to check to see if (a) is continued after other parts for evidence of this.

Condone the lack of $\mathrm{f}'(x)=$ on the left hand side or allow the use of $\frac{\mathrm{d}y}{\mathrm{d}x}$ or y' instead.

Alternative (a) attempt using the triple product rule

e.g. $$\frac{\mathrm{d}}{\mathrm{d}x}\left(7x\mathrm{e}^{x}(\mathrm{e}^{3x}-2)^{-\frac{1}{2}}\right)=7\mathrm{e}^{x}(\mathrm{e}^{3x}-2)^{-\frac{1}{2}}+7x\mathrm{e}^{x}(\mathrm{e}^{3x}-2)^{-\frac{1}{2}}+7x\mathrm{e}^{x}\times\left(-\frac{1}{2}\right)\times3\mathrm{e}^{3x}(\mathrm{e}^{3x}-2)^{-\frac{3}{2}}$$

$$\Rightarrow\frac{\left(7\mathrm{e}^{x}+7x\mathrm{e}^{x}\right)(\mathrm{e}^{3x}-2)+7x\mathrm{e}^{x}\times\left(-\frac{1}{2}\right)\times3\mathrm{e}^{3x}}{(\mathrm{e}^{3x}-2)^{\frac{3}{2}}}=\frac{7\mathrm{e}^{x}\left(\mathrm{e}^{3x}-2+x\mathrm{e}^{3x}-2x-\frac{3}{2}x\mathrm{e}^{3x}\right)}{(\mathrm{e}^{3x}-2)^{\frac{3}{2}}}\Rightarrow\frac{7\mathrm{e}^{x}\left(\mathrm{e}^{3x}(2-x)-4x-4\right)}{2(\mathrm{e}^{3x}-2)^{\frac{3}{2}}}$$

M1: Attempts the product rule on $x\mathrm{e}^{x}\rightarrow ...x\mathrm{e}^{x}\pm...\mathrm{e}^{x}$ which may be seen within the expression $...\mathrm{e}^{x}(\mathrm{e}^{3x}-2)^{-\frac{1}{2}}\pm..x\mathrm{e}^{x}(\mathrm{e}^{3x}-2)^{-\frac{1}{2}}+...$ simplified or unsimplified.

A1: $k\left(x\mathrm{e}^{x}+\mathrm{e}^{x}\right)$ which may be seen within the expression $k\left(\mathrm{e}^{x}(\mathrm{e}^{3x}-2)^{-\frac{1}{2}}+x\mathrm{e}^{x}(\mathrm{e}^{3x}-2)^{-\frac{1}{2}}\right)+...$ simplified or unsimplified.

B1: $\left(-\frac{1}{2}\right)\times3\mathrm{e}^{3x}(\mathrm{e}^{3x}-2)^{-\frac{3}{2}}$ which may be seen within the expression $....+k\left(x\mathrm{e}^{x}\times\left(-\frac{1}{2}\right)\times3\mathrm{e}^{3x}(\mathrm{e}^{3x}-2)^{-\frac{3}{2}}\right)$ simplified or unsimplified.

dM1: A complete method using all three products (which may appear all on one line). Do not condone invisible brackets.

A1: As above in main scheme notes.

(b) Note that if they do not have values $A=-4,\ B=-4$ in (a) (which may be seen later) then maximum score is M1A0*

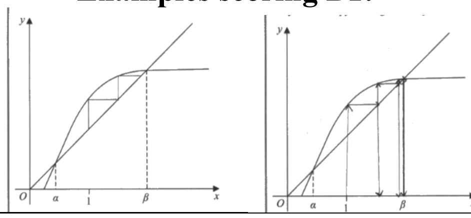
M1: Sets their $\mathrm{e}^{3x}(2-x)-4x-4$ equal to zero, collects terms in x on one side of the equation and non x terms on the other and attempts to factorise the side with x as a common factor. Condone sign slips only for this mark. Allow A and B to be used instead of $"-4"$ and $"-4"$

A1*: Achieves the given answer with no errors including invisible brackets. If they do not reach the printed answer then it is A0. If they subsequently write $x_{n+1} = \dfrac{2e^{3x_n} - 4}{e^{3x_n} + 4}$ then isw

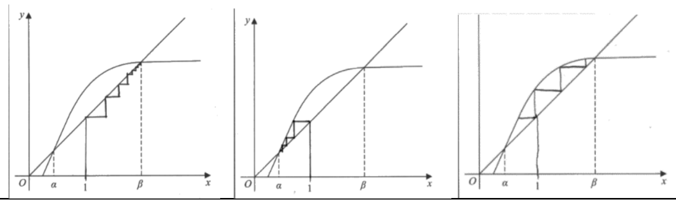
(c)

B1: Starting at $x_1 = 1$ look for at least 2 sets of vertical and horizontal lines drawn (may be dashes) tending to β. Condone a lack of arrows on the lines but the sequence of lines should finish at the point of intersection where the root is. Condone the initial vertical line not starting from the x-axis. Mark the intention to draw horizontal and vertical lines. If they have any lines to the left of $x = 1$ this is B0. If they use both diagrams and do not indicate which one they want marking, then the "copy of Diagram 1" should be marked.

Examples scoring B1:



Examples scoring B0:



(d)(i)

M1: Substitutes 1 into the iterative formula. The values embedded in the formula is sufficient for this mark. May be implied by awrt 1.50

A1: awrt 1.502 isw

(d)(ii)

dB1: 1.968 cao (which can only be scored if M1 is scored in (d)(i))

SC: If (d)(i) is rounded to 1.50 then allow 1.97 in (d)(ii) to score M1A0dB1 for (d)

(e)

M1: Attempts to substitute $x = 0.4315$ and 0.4325 into a suitable function and gets one value correct (rounded or truncated to 1sf). It is allowable to use a tighter interval that contains the root 0.4317388728…

If no function is stated then may be implied by their answers to e.g. $f'(0.4315)$, $f'(0.4325)$

You will need to check their calculation is correct.

Other possible functions include:

- $h(x) = x - \dfrac{2e^{3x} - 4}{e^{3x} + 4}$ (other way round to MS) $h(0.4315) = 0.0002974...$, $h(0.4325) = -0.0009479...$
- their $f'(x) = \pm\left(\dfrac{7e^{x}\left(e^{3x}(2-x) - 4x - 4\right)}{2(e^{3x} - 2)^{\frac{3}{2}}}\right)$

  (If correct A and B then $f'(0.4315) = \mp 0.005789...$, $f'(0.4325) = \pm 0.01831...$)
- their $g(x) = \pm\left(e^{3x}(2-x) - 4x - 4\right)$

  (If correct A and B then $g(0.4315) = \mp 0.002275...$, $g(0.4325) = \pm 0.007261...$)

A1: Requires

- Both calculations correct (rounded or truncated to 1sf)
- A statement that there is a change in sign and that their **function** is continuous (must refer to the function used for the substitution (which is not $f(x)$)

  Accept equivalent statements for $f'(0.4315) < 0$, $f'(0.4325) > 0$ e.g. $f'(0.4315) \times f'(0.4325) < 0$, "one negative one positive". A minimum is "change of sign and continuous" but do **not** allow this mark if the comment about continuity is clearly incorrect e.g. "because x is continuous" or "because the interval is continuous"
- A minimal conclusion e.g. "hence $\alpha = 0.432$", "so rounds to 0.432". Do not allow "hence root"

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