2025/2026

Rate of change

PURE 4

MATHS WITH DILAN

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Q1

The volume of a spherical balloon of radius r cm is V cm^3, where $V = \frac{4}{3}\pi r^3$.

(a) Find $\frac{\mathrm{d}V}{\mathrm{d}r}$.

(1)

The volume of the balloon increases with time t seconds according to the formula

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{1000}{(2t+1)^2},\ t \geq 0.$$

(b) Using the chain rule, or otherwise, find an expression in terms of r and t for $\frac{\mathrm{d}r}{\mathrm{d}t}$.

(2)

(c) Given that $V = 0$ when $t = 0$, solve the differential equation $\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{1000}{(2t+1)^2}$, to obtain V in terms of t.

(4)

(d) Hence, at time $t = 5$,

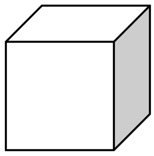
(i) find the radius of the balloon, giving your answer to 3 significant figures,

(3)

(ii) show that the rate of increase of the radius of the balloon is approximately 2.90×10^{-2} cm s^{-1}.

(2)

Q2



At time t seconds the length of the side of a cube is x cm, the surface area of the cube is S cm^2, and the volume of the cube is V cm^3.

The surface area of the cube is increasing at a constant rate of 8 cm^2 s^{-1}.

Show that

(*a*) $\frac{dx}{dt} = \frac{k}{x}$, where k is a constant to be found,

(4)

(*b*) $\frac{dV}{dt} = 2V^{\frac{1}{3}}$.

(4)

Given that $V = 8$ when $t = 0$,

(*c*) solve the differential equation in part (*b*), and find the value of t when $V = 16\sqrt{2}$.

(7)

Q3

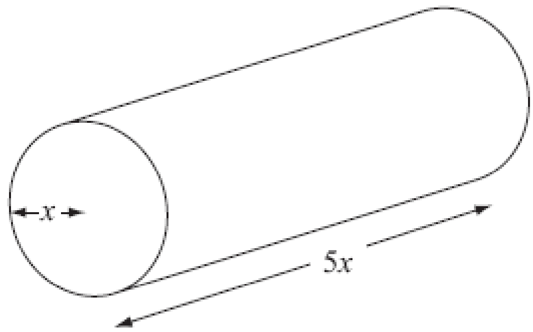


Figure 2

Figure 2 shows a right circular cylindrical metal rod which is expanding as it is heated. After t seconds the radius of the rod is x cm and the length of the rod is $5x$ cm.

The cross-sectional area of the rod is increasing at the constant rate of 0.032 $\text{cm}^2\,\text{s}^{-1}$.

(*a*) Find $\frac{dx}{dt}$ when the radius of the rod is 2 cm, giving your answer to 3 significant figures.

(4)

(*b*) Find the rate of increase of the volume of the rod when $x = 2$.

(4)

Q4

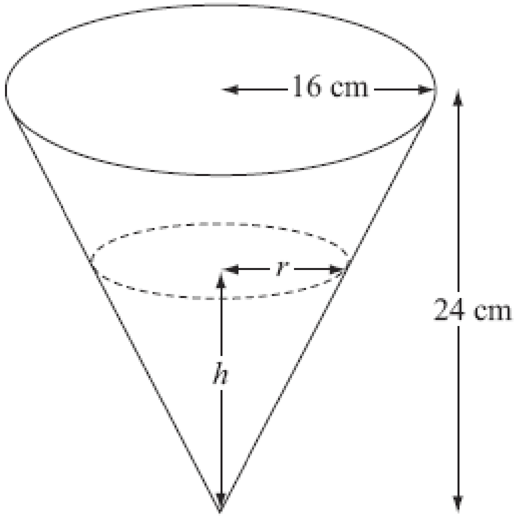


Figure 2

A container is made in the shape of a hollow inverted right circular cone. The height of the container is 24 cm and the radius is 16 cm, as shown in Figure 2. Water is flowing into the container. When the height of water is h cm, the surface of the water has radius r cm and the volume of water is V cm^3.

(a) Show that $V = \dfrac{4\pi h^3}{27}$.

(2)

[*The volume V of a right circular cone with vertical height h and base radius r is given by the formula* $V = \dfrac{1}{3}\pi r^2 h$.]

Water flows into the container at a rate of 8 cm^3 s^{-1}.

(b) Find, in terms of π, the rate of change of h when $h = 12$.

(5)

Q5

The area A of a circle is increasing at a constant rate of 1.5 cm^2 s^{-1}. Find, to 3 significant figures, the rate at which the radius r of the circle is increasing when the area of the circle is 2 cm^2.

(5)

Q6

The current, I amps, in an electric circuit at time t seconds is given by

$$I = 16 - 16(0.5)^t, \qquad t \geq 0.$$

Use differentiation to find the value of $\dfrac{dI}{dt}$ when $t = 3$.

Give your answer in the form $\ln a$, where a is a constant.

(5)

Q7

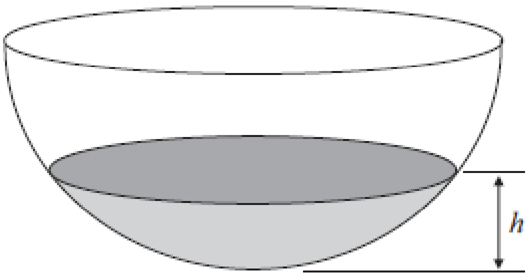


Figure 1

A hollow hemispherical bowl is shown in Figure 1. Water is flowing into the bowl.

When the depth of the water is h m, the volume V m^3 is given by

$$V = \frac{1}{12}\pi h^2(3 - 4h), \qquad 0 \leq h \leq 0.25.$$

(a) Find, in terms of π, $\dfrac{dV}{dh}$ when $h = 0.1$.

(4)

Water flows into the bowl at a rate of $\dfrac{\pi}{800}$ m^3 s^{-1}.

(b) Find the rate of change of h, in m s^{-1}, when $h = 0.1$.

(2)

Q8

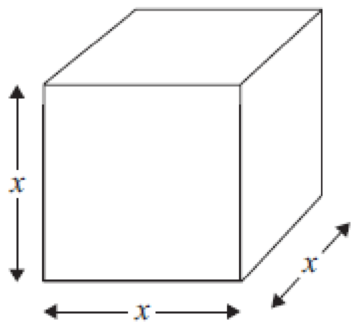


Figure 1

Figure 1 shows a metal cube which is expanding uniformly as it is heated.

At time t seconds, the length of each edge of the cube is x cm, and the volume of the cube is V cm^3.

(*a*) Show that $\dfrac{dV}{dx} = 3x^2$.

(1)

Given that the volume, V cm^3, increases at a constant rate of 0.048 cm^3 s^{-1},

(*b*) find $\dfrac{dx}{dt}$ when $x = 8$,

(2)

(*c*) find the rate of increase of the total surface area of the cube, in cm^2 s^{-1}, when $x = 8$.

(3)

ANSWERS

Q1

(a) $4\pi r^2$ (b) $\dfrac{1000}{4\pi r^2(2t+1)^2}$ (c) $500(1-\dfrac{1}{2t+1})$ (d) (i) 4.77

Q2

(c) $\dfrac{75}{\ln 2}$

Q3

(a) 0.00254 cm s^{-1} (b) 0.48 cm^3 s^{-1}

Q4

(b) $\dfrac{1}{8\pi}$

Q5

0.299 cm s^{-1}

Q6

$\ln 4$

Q7

(a) 0.04π (b) $\frac{1}{32}$

Q8

(b) $0.000\,25$ cm s^{-1} (c) 0.024 cm^2 s^{-1}