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Western Province Educational Department

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, **2024**
General Certificate of Education (Adv. Level) Examination, 2024

සංයුක්ත ගණිතය II Combined Mathematics II	**10 E II**	**2024.11.01 / 08.30 - 11.40**	
පැය තුනයි *Three hours*	අමතර කියවීම් කාලය - මිනිත්තු 10 යි Additional Reading Time - 10 minutes		

additional reading time to go through the question paper, select the questions and decide on the questions that you give priority in answering.

Index Number							

Instructions:

* *This question paper consists of two parts.*
  Part A (Questions 01 - 10) *and* **Part B** (Questions 11 - 17)
* **Part A:**
  *Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
* **Part B:**
  *Answer **five** questions only. Write your answers on the sheets provided.*
* *At the end of the time allotted, tie the answer scripts of the two parts together so that* **Part A** *is on top of* **Part B** *and hand them over to the supervisor.*
* *You are permitted to remove only* **Part B** *of the question paper from the Examination Hall.*

For Examiners' Use only

(10) Combined Mathematics II		
Part	**Question No.**	**Marks**
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	
	Percentage	

Final Marks

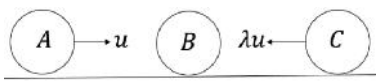
In Numbers	
In Words	

Code Numbers

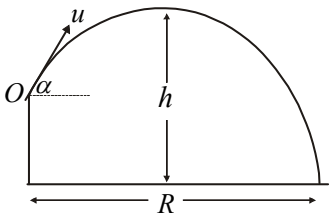
Marking Examiner	
Checked by: 1	
2	
Supervised by:	

Part A

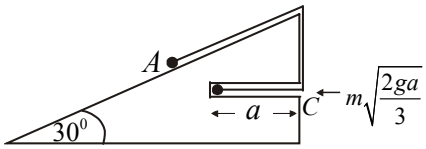
01. Three equal particles A, B and C each of mass m, placed in a horizontal line. A and C are projected towards B with velocity u and λu respectively. First, A and B **collide,** and then B and C collide and coalesce. Just after the second collision, the combined particle BC moves with velocity $\frac{u}{4}$. show that the coefficient of restitution between A and B is 2λ.



02. A particle is projected from a point O at a height $\frac{h}{2}$ from a horizontal floor with initial velocity u and at an angle $\alpha\left(0<\alpha<\frac{\pi}{2}\right)$ to the **horizontal.** The maximum height of the particle is h and the horizontal range on the ground is R. Show that $h-2R\tan\alpha+\frac{gR^2}{u^2-gh}=0$.



03. A particle of mass m is placed at a point B in a smooth horizontal hole in a fixed wedge. A particle of mass $2m$ is placed on the smooth inclined plane of inclination 30^0 to the horizontal and attached the both particles by a light inextensible string as shown in the figure. When the system released form rest, find the common acceleration of the particles.



When the particle of mass m comes to point C, it is given and impulse $m\sqrt{\frac{2ga}{3}}$ horizontally, then show that the particle of mass m became instantaneous rest.

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04. A car with mass M kg and maximum power H kW travels on a straight horizontal road with velocity v, against a constant resistance with 50% of its power. Then car travels up along an inclined plane with inclination α to the horizontal. Find the acceleration of the car when it travels with velocity v.

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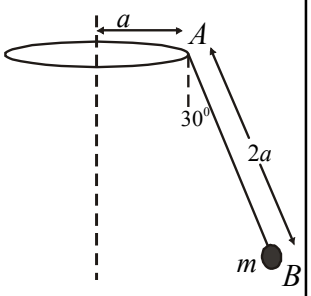
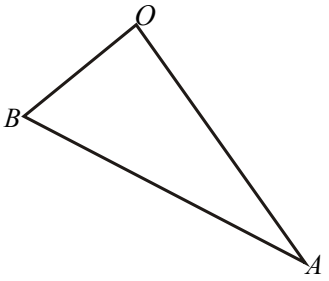
05. A particle of mass m is fastened to the end B of light rod AB, which is attached to a point A on the circumference of a horizontal circular disc of radius a, as shown in the figure. When the disc rotates around its axis with uniform angular velocity, the particle describes a horizontal circle such that the inclination of the rod to the vertical is 30^0. Find the tension in the light rod and show that the angular velocity is $\sqrt{\frac{g}{2\sqrt{3a}}}$.



06. In the usual notation, let $2\mathbf{i}+\alpha\mathbf{j}$ and $\beta\mathbf{i}+\mathbf{j}$ be the position vectors of two points A and B respectively, with respect to a fixed origin O. Also, let C be the point such that $OACB$ is a rhombus. Show that $\beta^2-\alpha^2=3$.

07. Two inelastic strings OA and OB of length $12a$ and $5a$ respectively, attached to a fixed point O and the ends A and B attached to the ends of non-uniform rod AB of length $13a$. If the center of gravity of the rod lies on the rod such that $AG : GC = 2 : 1$, then find the inclination of the rod to the horizontal.



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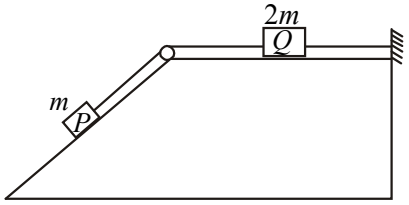
08. A particle P of mass m is placed on a fixed rough inclined plane of inclination $\tan^{-1}\left(\frac{3}{4}\right)$ to the horizontal. It is connected to another particle Q with a mass $2m$ by a light inelastic string as in the adjacent figure.



The coefficient of friction between P and inclined plane is $\frac{1}{3}$. An elastic string with a natural length l and elastic modulus λ is attached to a fixed point on the horizontal plane and connected to Q. When the system is at equilibrium, the extension of the elastic string is l.

Show that $\frac{mg}{3} \le \lambda \le \frac{13mg}{15}$.

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09. Let A and B be two independent events. The probability that exactly one of them occurs is $\frac{11}{25}$ and the probability of non of them occurring is $\frac{2}{25}$. What is the probability of **non** occurrence of event A.

10. The values of six observations in ascending order are 1, 5, x, y, z, 12 $\left(x, y, z \in \mathbb{Z}^+\right)$ there is no any mode and the mean and the median equal to 7. Find the values of x, y and z and the variance of six observations.

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අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, **2024**
General Certificate of Education (Adv. Level) Examination, 2024

සංයුක්ත ගණිතය II Combined Mathematics II	**10 E II**	**2024.11.01 / 08.30 - 11.40**

Part B

■ Answer **only five** Questions.

11. (*a*) A and B are two points on a straight path. B car P starts to move from rest at $t = 0$ and moves towards B with constant acceleration f. After time $t_0 \left(< \frac{u}{f} \right)$, another Q passes the point A, moves towards B with uniform velocity u. P and Q just collide.

It is given that due to the collision, the velocity of P **increases** by an amount $\frac{u}{2}$ and the velocity of Q **decreases** by an amount $\frac{u}{2}$. In the subsequent motion, P moves with constant acceleration f and Q moves with constant retardation $2f$ and becomes rest at B.

Draw the velocity time graph for the motions of P and Q in the same diagram. Hence,

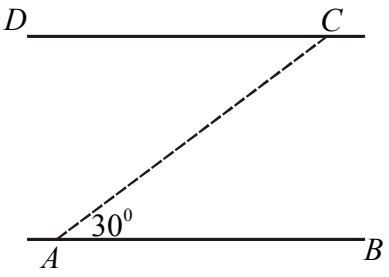
Show that

(i) $t_0 = \frac{u}{2f}$

(ii) $AB = \frac{9u}{16f}$

(iii) When Q become rest, P at a distance $\frac{11u^2}{32f}$ from B.

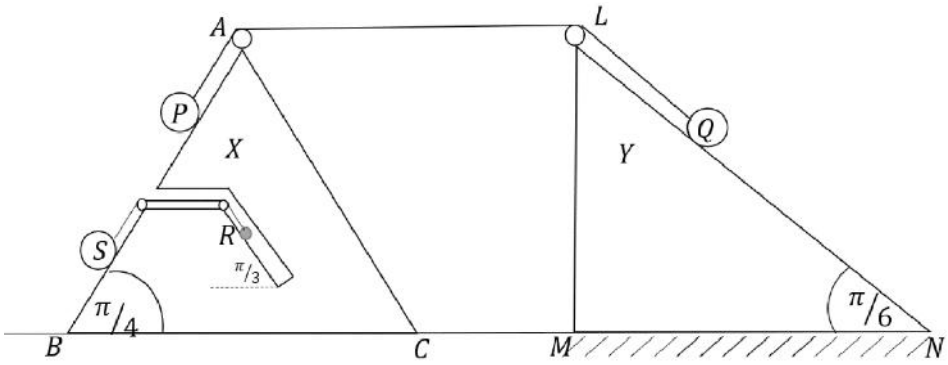
(*b*) A river with parallel straight banks of width d, flows with uniform velocity u, A, B, C and D are located on the banks forming a rectangle $ABCD$ as shown in the figure.



A small ship, which can move with velocity $\frac{u}{2}$ relative to the river moves along AC from A. In the same instance, two boats B_1 and B_2 start at B and D respectively with same velocity v along a straight paths.

If the boats B_1 and B_2 meet the ship at a time t_1 and t_2 then, Find t_1 and t_2

12. (*a*)

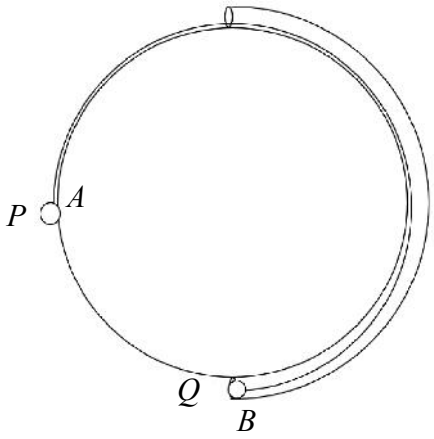


The triangles ABC and LMN in the figure represent the vertical cress section through the center of gravity of two uniform smooth wedges X and Y with the faces BC and MN are in contact with smooth horizontal floor.

The wedge X of mass $5m$ free to move along the smooth floor and Y is fixed. A particle R of mass m is placed on a hole in the wedge X as shown in the figure and it is connected to the particle S on AB by a light inextensible string.

The lines AB and LN are lines of greatest slope of the faces containing it. Two particles P and Q of masses $2m$ and $3m$, placed on AB and LN respectively, are connected by a light inextensible string which passes over two small smooth pulleys at the vertices A and L. The system is released from rest with the strings taut. **Obtain sufficient equations** to determine the accelerations of four particles and the wedge X.

(*b*) A small semicircular narrow tube is connected to a semicircular rail of radius a and fixed in a vertical plane as shown in the figure. A particle P of mass $2m$ is placed at point A on horizontal diameter and a particle Q of mass is placed at the lowest point B of the tube. P and Q are connected by a light inextensible string of length $\frac{3\pi a}{2}$.



The system is released from rest with string taut. Show that the speed v of the particles when OQ has turned though an angle $\theta\left(0<\theta<\frac{\pi}{2}\right)$ is given by $v^2=\frac{2ga}{3}(\theta+\cos\theta-1)$.

Find the reaction on the particle P from the tube. In the subsequent motion, show that one instant the reaction between Q and tube. Alos find the tention of the string at that instant.

13. A particle of mass $2m$ hangs in equilibrium at one end of a light elastic string, of natural length l, whose other end is attached to a fixed point O. In the vertical equilibrium position, the extension in the string is l. Show that the modulus of the elasticity of the string is $2mg$.

Now the mass $2m$ is removed and a particle P of mass m is attached to the same end and another particle Q of mass freely hung at P by a **light inextensible** string of length l. When the particles are in equilibrium, particle Q is pulled down at a distance l, by applying a force on it. when the strings taut and the extension the string is $x+l$, show that the equation of the motion of particle P is given by $\ddot{x}+\frac{g}{l}x=0$.

By using the equation $\dot{X}^2=\omega^2\left(c^2-X^2\right)$, find the amplitude c of the motion.

The string PQ cuts when the particle reaches the point A, which is at a distance l vertically below O. In the subsequent motion, show that the equation of the motion of the particle is given by $\ddot{y}+\frac{2g}{l}\left(y-\frac{l}{2}\right)=0$.

Assuming that the solution of the above equation is of the form $y=\frac{1}{2}+\alpha\cos\omega t+\beta\sin\omega t$, find the values of the constants α, β and ω.

Hence, find the centre and amplitude of the simple harmonic motion performed by the particle from A to B. Where B is the lowest point of the motion. Also find the velocity of P at point D, vertically $\frac{7l}{4}$ below O. Show that the total time taken to reached D is $\frac{\pi}{3}\sqrt{\frac{l}{g}}\left(3+\sqrt{2}\right)$.

14. (*a*) Let $OABC$ is a pralleogram and the point D lies on AC such that $AD:DC=4:3$. The position vectors of points A and B with respect to the origin O are $\lambda\mathbf{a}$ and $\mathbf{b}$. Where $\lambda>0$.

Express $\overrightarrow{OC}$ and $\overrightarrow{BD}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and λ.

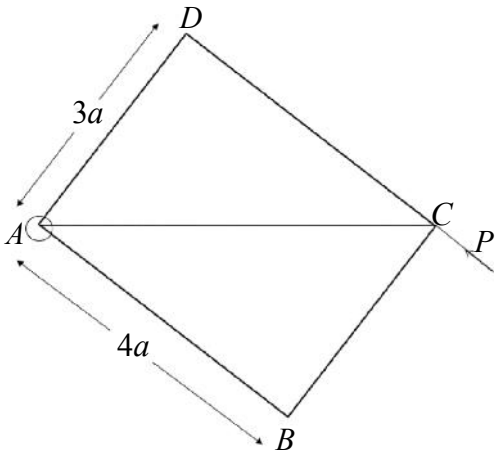
E be the mid point of AB. Now, let $\overrightarrow{CE}$ is perpendicular to $\overrightarrow{BD}$.

Show that $3|\mathbf{a}|^2\lambda^2+8(\mathbf{a}\cdot\mathbf{b})\lambda-3|\mathbf{b}|^2=0$. Further, it is given that $\mathbf{a}$ and $\mathbf{b}$ are unit vectors and $A\hat{O}B=\frac{\pi}{3}$, then find the value of λ.

(*b*) The position vectors of points A, B and C with respect to the origin O are $\mathbf{i}+\mathbf{j}$, $\mathbf{i}+5\mathbf{j}$ and $3\mathbf{i}+\left(1+2\sqrt{3}\right)\mathbf{j}$ respectively. The forces αF, βF and γF act respectively along the sides $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{AC}$. Express these three forces in the form $a\mathbf{i}+b\mathbf{j}$. It is given that the force system reduces to a couple. Show that $\beta=\alpha$ and $\gamma=-3\alpha$. Find the moment of the couple in terms of α.

It is given $\alpha=5$, $\beta=2$ and $\gamma=3$. Show that the system is not in equilibrium. Find the magnitude of the resultant and equation of the line of action of the resultant.

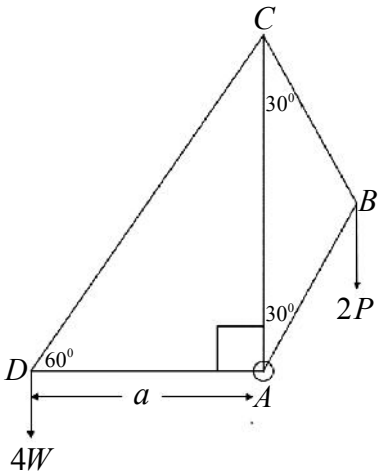
15. (*a*) Four uniform rods AB, BC, CD and DA, **each of weight** W, but in differing in lengths are smoothly joined at the ends A, B, C and D form a rectangle by a light rod joining A and C. $AB = CD = 4a$ and $AD = BC = 3a$. The end A is smoothly hinged to a fixed point such that the rectangle can rotate freely in a vertical plane. The rods are kept in equilibrium in avertical plane by applying a P at C along CD with AC horizontal and B below at AC.



Show that $P = \frac{10W}{3}$ and find the horizontal and vertical components of the reaction at D along the directions AD and CD. Also find the thrust in the light rod.

(*b*) The diagram shows that a framework consists of five light rods AB, BC, AD, AC and CD. $AD = a$, $A\hat{D}C = 60^0$ and $A\hat{C}B = C\hat{A}B = 30^0$. A weights $4W$ and $2P$ are freely hung at D and B respectively. It is smoothly hinged at A and kept in equilibrium in a vertical plane as shown in the figure. Find the value P of in terms of W.

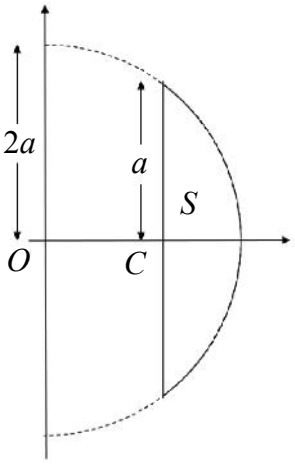
Draw a stress diagram using Bow's notation and **hence**, find the stresses in the rods stating whether they are tensions or thrusts.



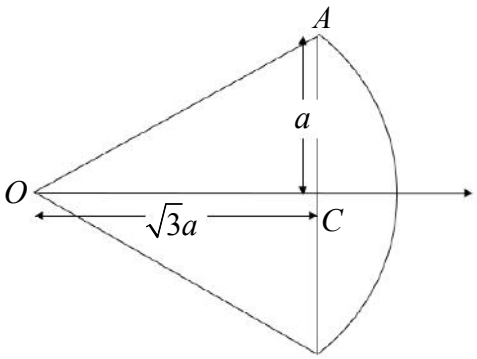
16. Show that the centre of mass ofa uniform solid right circular cone of base radius r and height h is at a distance $\frac{h}{4}$ from the centre of the base.

Show by the integration the centre of mass of the object S, obtained by cutting a uniform solid hemisphere with centre O and radius $2a$, perpendicular to its axis

The object S is made by cutting a part perpendicular to its axis from a solid hemisphere of radius $2a$ and centreof base O as shown in the figure. The centre of S is C and the radius a. Show that the center of mass of S lies on its symmetrical axis at a distance $\frac{3a}{4\left(16-6\sqrt{3}\right)}$ from O.



The object shown in the adjacent figure, a right circular cone made by same material with center and height $\sqrt{3}a$ rigidly jointed as coincide with the base of S. Show that the center of mass of compound object lies on its symmetric axis, at a distance $\frac{3a}{8}\left(2+\sqrt{3}\right)$ from O.



When the compound object is freely hung at A, find the weight of load should added at the end B, so that the axis is **horizontal.**

17. (*a*) Let A and B be two events in a sample space. In usual notation, if A' and B' are independent, then show that

I. A and B

II. A' and B are independent.

(*b*) Red and white balls, equal in size are put in a three boxes as shown in the table. By rolling a fair die, the boxes are selected according to the face value of the die.

Box	No. of red balls	No. of White balls	Face value of the die
A	3	5	1, 2, 3
B	4	4	4, 5
C	5	3	6

Find the probability that, by rolling the die and selecting a box, the ball chosen is white.
It is given a white ball is chosen, find the probability that it is chosen from box B.

(*c*) When the height of 100 students of a school was measured to the nearest centimeter, it was obtained as follows.

Height (cm)	150-154	155-159	160-164	165-169	170-174
No. of Students	9	17	35	23	16

Estimate the mean μ and the standard deviation σ of the distribution given in this table. Also, estimate the coefficient of skewness k defined by $k=\frac{\mu-M_0}{\sigma}$, where M_0 is the mode.