P1 January 2001

(a) Prove, by completing the square, that the roots of the equation $x^2 + 2kx + c = 0$, where k and c are constants, are $-k \pm \sqrt{(k^2 - c)}$.

(4 marks)

The equation $x^2 + 2kx \pm 81 = 0$ has equal roots.

(b) Find the possible values of k.

(2 marks)

P1 June 2002

Given that $f(x) = 15 - 7x - 2x^2$,

(a) find the coordinates of all points at which the graph of $y = f(x)$ crosses the coordinate axes.

(3)

(b) Sketch the graph of $y = f(x)$.

(2)

P1 June 2002

(a) By completing the square, find in terms of k the roots of the equation

$$x^2 + 2kx - 7 = 0.$$

(4)

(b) Prove that, for all values of k, the roots of $x^2 + 2kx - 7 = 0$ are real and different.

(2)

(c) Given that $k = \sqrt{2}$, find the exact roots of the equation.

(2)

P1 January 2004

$$f(x) = x^2 - kx + 9, \text{ where } k \text{ is a constant.}$$

(*a*) Find the set of values of k for which the equation $f(x) = 0$ has no real solutions.

(4)

Given that $k = 4$,

(*b*) express $f(x)$ in the form $(x - p)^2 + q$, where p and q are constants to be found,

(3)

(*c*) write down the minimum value of $f(x)$ and the value of x for which this occurs.

(2)

P1 June 2005

$$x^2 - 8x - 29 \equiv (x + a)^2 + b,$$

where a and b are constants.

(a) Find the value of a and the value of b.

(3)

(b) Hence, or otherwise, show that the roots of

$$x^2 - 8x - 29 = 0$$

are $c \pm d\sqrt{5}$, where c and d are integers to be found.

(3)

P1 January 2006

$$x^2 + 2x + 3 \equiv (x + a)^2 + b.$$

(a) Find the values of the constants a and b.

(2)

(b) In the spaces provided below, sketch the graph of $y = x^2 + 2x + 3$, indicating clearly the coordinates of any intersections with the coordinate axes.

(3)

(c) Find the value of the discriminant of $x^2 + 2x + 3$. Explain how the sign of the discriminant relates to your sketch in part (b).

(2)

The equation $x^2 + kx + 3 = 0$, where k is a constant, has no real roots.

(d) Find the set of possible values of k, giving your answer in surd form.

(4)

January 2005

Given that

$$\mathrm{f}(x) = x^2 - 6x + 18, \quad x \geq 0,$$

(a) express f(x) in the form $(x - a)^2 + b$, where a and b are integers.

(3)

The curve C with equation $y = \mathrm{f}(x)$, $x \geq 0$, meets the y-axis at P and has a minimum point at Q.

(b) Sketch the graph of C, showing the coordinates of P and Q.

(4)

The line $y = 41$ meets C at the point R.

(c) Find the x-coordinate of R, giving your answer in the form $p + q\sqrt{2}$, where p and q are integers.

(5)

June 2005

$$x^2 - 8x - 29 \equiv (x + a)^2 + b,$$

where a and b are constants.

(a) Find the value of a and the value of b.

(3)

(b) Hence, or otherwise, show that the roots of

$$x^2 - 8x - 29 = 0$$

are $c \pm d\sqrt{5}$, where c and d are integers to be found.

(3)

January 2006

$$x^2 + 2x + 3 \equiv (x + a)^2 + b.$$

(a) Find the values of the constants a and b.

(2)

(b) Sketch the graph of $y = x^2 + 2x + 3$, indicating clearly the coordinates of any intersections with the coordinate axes.

(3)

(c) Find the value of the discriminant of $x^2 + 2x + 3$. Explain how the sign of the discriminant relates to your sketch in part (b).

(2)

The equation $x^2 + kx + 3 = 0$, where k is a constant, has no real roots.

(d) Find the set of possible values of k, giving your answer in surd form.

(4)

January 2010

$$\text{f}(x) = x^2 + 4kx + (3 + 11k), \quad \text{where } k \text{ is a constant.}$$

(a) Express f(x) in the form $(x + p)^2 + q$, where p and q are constants to be found in terms of k.

(3)

Given that the equation f(x) = 0 has no real roots,

(b) find the set of possible values of k.

(4)

Given that $k = 1$,

(c) sketch the graph of $y = \text{f}(x)$, showing the coordinates of any point at which the graph crosses a coordinate axis.

(3)

June 2010

(*a*) Show that $x^2 + 6x + 11$ can be written as

$$(x + p)^2 + q,$$

where p and q are integers to be found.

(2)

(*b*) Sketch the curve with equation $y = x^2 + 6x + 11$, showing clearly any intersections with the coordinate axes.

(2)

(*c*) Find the value of the discriminant of $x^2 + 6x + 11$.

(2)

June 2012

$$4x - 5 - x^2 = q - (x + p)^2,$$

where p and q are integers.

(*a*) Find the value of p and the value of q.

(3)

(*b*) Calculate the discriminant of $4x - 5 - x^2$.

(2)

(*c*) Sketch the curve with equation $y = 4x - 5 - x^2$, showing clearly the coordinates of any points where the curve crosses the coordinate axes.

(3)

January 2013

$$4x^2 + 8x + 3 \equiv a(x + b)^2 + c.$$

(*a*) Find the values of the constants a, b and c.

(3)

(*b*) Sketch the curve with equation $y = 4x^2 + 8x + 3$, showing clearly the coordinates of any points where the curve crosses the coordinate axes.

(4)

ANSWERS

January 2005

(*a*) $(x-3)^2+9$ (*b*) P (0, 18); Q (3, 9) (*c*) $3+4\sqrt{2}$

June 2005

(*a*) $a=-4$; $b=-45$

January 2006

(*a*) $a=1$; $b=2$ (*c*) -8 (*d*) $-\sqrt{12} < k < \sqrt{12}$

January 2010

(*a*) $(x+2k)^2-4k^2+(3+11k)$ (*b*) $-\frac{1}{4}<k<3$

June 2010

N/A

June 2012

(*a*) $p=-1$, $q=2$ (*c*) $(0, -5)$