P2 January 2002

$$f(x) = \left(1 + \frac{x}{k}\right)^n, \qquad k, n \in \mathbb{N}, \qquad n > 2.$$

Given that the coefficient of x^3 is twice the coefficient of x^2 in the binomial expansion of f(x),

(a) prove that $n = 6k + 2$.

(3)

Given also that the coefficients of x^4 and x^5 are equal and non-zero,

(b) form another equation in n and k and hence show that $k = 2$ and $n = 14$.

(4)

Using these values of k and n,

(c) expand f(x) in ascending powers of x, up to and including the term in x^5. Give each coefficient as an exact fraction in its lowest terms.

(4)

P2 June 2002

(a) Write down the first four terms of the binomial expansion, in ascending powers of x, of

$(1 + 3x)^n$, where $n > 2$.

(2)

Given that the coefficient of x^3 in this expansion is ten times the coefficient of x^2,

(b) find the value of n,

(2)

(c) find the coefficient of x^4 in the expansion.

(2)

P2 January 2003

The first three terms in the expansion, in ascending powers of x, of $(1 + px)^n$, are $1 - 18x + 36p^2x^2$. Given that n is a positive integer, find the value of n and the value of p.

(7)

P2 June 2003

The expansion of $(2 - px)^6$ in ascending powers of x, as far as the term in x^2, is

$$64 + Ax + 135x^2.$$

Given that $p > 0$, find the value of p and the value of A.

(7)

P2 November 2003

(*a*) Write down the first 4 terms of the binomial expansion, in ascending powers of x, of

$$(1 + ax)^n, \; n > 2.$$

(2)

Given that, in this expansion, the coefficient of x is 8 and the coefficient of x^2 is 30,

(*b*) calculate the value of n and the value of a,

(4)

(*c*) find the coefficient of x^3.

(2)

P2 June 2004

For the binomial expansion, in descending powers of x, of

$$\left(x^3 - \frac{1}{2x}\right)^{12},$$

(*a*) find the first 4 terms, simplifying each term.

(5)

(*b*) Find, in its simplest form, the term independent of x in this expansion.

(3)

P2 November 2004

(*a*) Find the first four terms, in ascending powers of x, in the binomial expansion of $\left(k + \frac{x}{2}\right)^5$, where k is a constant.

(2)

Given that the third term of this series is $540x^2$,

(*b*) show that $k = 6$,

(2)

(*c*) find the coefficient of x^3.

(2)

C2 January 2005

Find the first three terms, in ascending powers of x, of the binomial expansion of $(3 + 2x)^5$, giving each term in its simplest form.

(4)

C2 June 2005

(a) Write down the first three terms, in ascending powers of x, of the binomial expansion of $(1 + px)^{12}$, where p is a non-zero constant.

(2)

Given that, in the expansion of $(1 + px)^{12}$, the coefficient of x is $(-q)$ and the coefficient of x^2 is $11q$,

(b) find the value of p and the value of q.

(4)

P2 January 2006

(a) Write down the binomial expansion, in ascending powers of x, of $(1 + 6x)^4$, giving each coefficient as an integer.

(3)

(b) Use your binomial expansion to find the exact value of 601^4.

(2)

C2 January 2006

(a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(1 + px)^9,$$

where p is a constant.

(2)

The first 3 terms are 1, $36x$ and qx^2, where q is a constant.

(b) Find the value of p and the value of q.

(4)

C2 June 2006

Find the first 3 terms, in ascending powers of x, of the binomial expansion of $(2 + x)^6$, giving each term in its simplest form.

(4)

C2 January 2007

(*a*) Find the first 4 terms, in ascending powers of x, of the binomial expansion of $(1 - 2x)^5$. Give each term in its simplest form.

(4)

(*b*) If x is small, so that x^2 and higher powers can be ignored, show that

$$(1 + x)(1 - 2x)^5 \approx 1 - 9x.$$

(2)

C2 June 2007

(a) Find the first four terms, in ascending powers of x, in the binomial expansion of $(1 + kx)^6$, where k is a non-zero constant.

(3)

Given that, in this expansion, the coefficients of x and x^2 are equal, find

(*b*) the value of k,

(2)

(*c*) the coefficient of x^3.

(1)

C2 January 2008

(*a*) Find the first 4 terms of the expansion of $\left(1+\frac{x}{2}\right)^{10}$ in ascending powers of x, giving each term in its simplest form.

(4)

(*b*) Use your expansion to estimate the value of $(1.005)^{10}$, giving your answer to 5 decimal places.

(3)

C2 June 2008

(*a*) Find the first 4 terms, in ascending powers of x, of the binomial expansion of $(1 + ax)^{10}$, where a is a non-zero constant. Give each term in its simplest form.

(4)

Given that, in this expansion, the coefficient of x^3 is double the coefficient of x^2,

(*b*) find the value of a.

(2)

C2 January 2009

Find the first 3 terms, in ascending powers of x, of the binomial expansion of $(3 - 2x)^5$, giving each term in its simplest form.

(4)

C2 June 2009

(a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(2 + kx)^7$$

where k is a constant. Give each term in its simplest form.

(4)

Given that the coefficient of x^2 is 6 times the coefficient of x,

(b) find the value of k.

(2)

C2 January 2010

Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(3 - x)^6$$

and simplify each term.

(4)

C2 June 2010

(a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of $(1 + ax)^7$, where a is a constant. Give each term in its simplest form.

(4)

Given that the coefficient of x^2 in this expansion is 525,

(b) find the possible values of a.

(2)

C2 January 2011

Given that $\binom{40}{4} = \frac{40!}{4!b!}$,

(a) write down the value of b.

(1)

In the binomial expansion of $(1 + x)^{40}$, the coefficients of x^4 and x^5 are p and q respectively.

(b) Find the value of $\frac{q}{p}$.

(3)

C2 June 2011

(a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(3 + bx)^5$$

where b is a non-zero constant. Give each term in its simplest form.

(4)

Given that, in this expansion, the coefficient of x^2 is twice the coefficient of x,

(b) find the value of b.

(2)

C2 January 2012

(a) Find the first 4 terms of the binomial expansion, in ascending powers of x, of

$$\left(1+\frac{x}{4}\right)^8,$$

giving each term in its simplest form.

(4)

(b) Use your expansion to estimate the value of $(1.025)^8$, giving your answer to 4 decimal places.

(3)

C2 June 2012

Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(2 - 3x)^5,$$

giving each term in its simplest form.

(4)

C2 January 2013

Find the first 3 terms, in ascending powers of x, in the binomial expansion of

$$(2 - 5x)^6.$$

Give each term in its simplest form.

(4)

ANSWERS:

C2 January 2005
$243 + 810x + 1080x^2$

C2 June 2005
(a) $1 + 12px + \dfrac{12\times 11}{2}(px)^2$ (b) $p = -2$, $q = 24$

C2 January 2006
(a) $1 + 9px + \dfrac{9}{2}(px)^2$ (b) $p = 4$; $q = 576$

C2 June 2006
$64 + 192x + 240x^2$

C2 January 2007
(a) $1 - 10x + 40x^2 - 80x^3 + ...$

C2 June 2007
(a) $1 + 6kx + \dfrac{6\times 5}{2}(kx)^2 + \dfrac{6\times 5\times 4}{3\times 2}(kx)^3$ (b) $\dfrac{2}{5}$ (c) $\dfrac{32}{25}$

C2 January 2008
(a) $1 + 5x + 11.25x^2 + 15x^3$ (b) 1.05114

C2 June 2008
(a) $1 + 10ax + 45(ax)^2 + 120(ax)^3 + ...$ (b) $a = 0.75$

C2 January 2009
$243 - 810x + 1080x^2$

C2 June 2009
(a) $128 + 448kx + 672k^2x^2$ (b) $k = 4$

C2 January 2010
$729 - 1458x + 1215x^2$

C2 June 2010
(a) $1 + 7ax + 21a^2x^2 + 35a^3x^3$ (b) 6.133

C2 January 2011

(*a*) 36 (*b*) 7.2

C2 June 2011

(*a*) $243 + 405bx + 270b^2x^2 + \ldots$ (*b*) $b = 3$

C2 January 2012

(*a*) $1 + 2x + \frac{7}{4}x^2 + \frac{7}{8}x^3$ (*b*) 1.2184

C2 June 2012

$32 - 240x + 720x^2$