2025
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Algebric methods

DILANMATHS
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P3 June 2001

$$f(x) \equiv ax^3 + bx^2 - 7x + 14, \text{ where } a \text{ and } b \text{ are constants.}$$

Given that when $f(x)$ is divided by $(x - 1)$ the remainder is 9,

(*a*) write down an equation connecting a and b.

(2)

Given also that $(x + 2)$ is a factor of $f(x)$,

(*b*) find the values of a and b.

(4)

P1 January 2002

$f(x) = x^3 - x^2 - 7x + c$, where c is a constant.

Given that $f(4) = 0$,

(*a*) find the value of c,

(2)

(*b*) factorise $f(x)$ as the product of a linear factor and a quadratic factor.

(3)

(*c*) Hence show that, apart from $x = 4$, there are no real values of x for which $f(x) = 0$.

(2)

P3 January 2002

$$f(x) = 4x^3 + 3x^2 - 2x - 6.$$

Find the remainder when $f(x)$ is divided by $(2x + 1)$.

(3)

P3 June 2002

$$f(x) = x^3 + ax^2 + bx - 10, \text{ where } a \text{ and } b \text{ are constants.}$$

When $f(x)$ is divided by $(x - 3)$, the remainder is 14.

When $f(x)$ is divided by $(x + 1)$, the remainder is -18.

(*a*) Find the value of a and the value of b.

(5)

(*b*) Show that $(x - 2)$ is a factor of $f(x)$.

(2)

P1 November 2002

(a) Using the factor theorem, show that $(x + 3)$ is a factor of

$$x^3 - 3x^2 - 10x + 24.$$

(2)

(b) Factorise $x^3 - 3x^2 - 10x + 24$ completely.

(4)

P3 January 2003

$$f(n) = n^3 + pn^2 + 11n + 9, \text{ where } p \text{ is a constant.}$$

(a) Given that $f(n)$ has a remainder of 3 when it is divided by $(n + 2)$, prove that $p = 6$.

(2)

(b) Show that $f(n)$ can be written in the form $(n + 2)(n + q)(n + r) + 3$, where q and r are integers to be found.

(3)

(c) Hence show that $f(n)$ is divisible by 3 for all positive integer values of n.

(2)

P3 June 2003

$$f(x) = px^3 + 6x^2 + 12x + q.$$

Given that the remainder when $f(x)$ is divided by $(x - 1)$ is equal to the remainder when $f(x)$ is divided by $(2x + 1)$,

(a) find the value of p.

(4)

Given also that $q = 3$, and p has the value found in part (a),

(b) find the value of the remainder.

(1)

P1 January 2004

$$f(x) = x^3 - 19x - 30.$$

(a) Show that $(x + 2)$ is a factor of $f(x)$.

(2)

(b) Factorise $f(x)$ completely.

(4)

P3 January 2004

$$f(x) = 6x^3 + px^2 + qx + 8, \text{ where } p \text{ and } q \text{ are constants.}$$

Given that f(x) is exactly divisible by ($2x – 1$), and also that when f(x) is divided by ($x – 1$) the remainder is –7,

(a) find the value of p and the value of q.

(6)

(b) Hence factorise f(x) completely.

(3)

P3 June 2004

$$f(x) = (x^2 + p)(2x + 3) + 3,$$

where p is a constant.

(a) Write down the remainder when f(x) is divided by ($2x + 3$).

(1)

Given that the remainder when f(x) is divided by ($x – 2$) is 24,

(b) prove that $p = –1$,

(2)

(c) factorise f(x) completely.

(4)

P1 November 2004

$$f(x) = x^3 + (p + 1)x^2 – 18x + q, \text{ where } p \text{ and } q \text{ are integers.}$$

Given that ($x – 4$) is a factor of f(x),

(a) show that $16p + q + 8 = 0$.

(3)

Given that ($x + p$) is also a factor of f(x), and that $p > 0$,

(b) show that $p^2 + 18p + q = 0$.

(3)

(c) Hence find the value of p and the corresponding value of q.

(5)

(d) Factorise f(x) completely.

(2)

P1 January 2005

$$f(x) = 2x^3 - x^2 + 2x - 16.$$

(*a*) Use the factor theorem to show that $(x - 2)$ is a factor of $f(x)$. **(2)**

Given that $f(x) = (x - 2)(2x^2 + bx + c)$,

(*b*) find the values of b and c. **(3)**

(*c*) Hence prove that $f(x) = 0$ has only one real solution. **(3)**

P3 January 2005

$f(x) = 2ax^3 - ax^2 - 3x + 7$,

where a is a constant.

Given that the remainder when $f(x)$ is divided by $(x + 2)$ is -3,

(*a*) find the value of a, **(3)**

(*b*) find the remainder when $f(x)$ is divided by $(2x - 1)$. **(2)**

P3 June 2005

A function f is defined as

$$f(x) = 2x^3 - 8x^2 + 5x + 6, \qquad x \in \mathbb{R}.$$

Using the remainder theorem, or otherwise, find the remainder when $f(x)$ is divided by

(*a*) $(x - 2)$, **(2)**

(*b*) $(2x + 1)$. **(2)**

(*c*) Write down a solution of $f(x) = 0$. **(1)**

P3 January 2006

$f(x) = 2x^3 – x^2 + ax + b$, where a and b are constants.

It is given that $(x – 2)$ is a factor of $f(x)$.

When $f(x)$ is divided by $(x + 1)$, the remainder is 6.

Find the value of a and the value of b.

(7)

C2 January 2005

$$f(x) = x^3 – 2x^2 + ax + b, \text{ where } a \text{ and } b \text{ are constants.}$$

When $f(x)$ is divided by $(x – 2)$, the remainder is 1.
When $f(x)$ is divided by $(x + 1)$, the remainder is 28.

(*a*) Find the value of a and the value of b.

(6)

(*b*) Show that $(x – 3)$ is a factor of $f(x)$.

(2)

C2 June 2005

(*a*) Use the factor theorem to show that $(x + 4)$ is a factor of $2x^3 + x^2 – 25x + 12$.

(2)

(*b*) Factorise $2x^3 + x^2 – 25x + 12$ completely.

(4)

C2 January 2006

$$f(x) = 2x^3 + x^2 – 5x + c, \text{ where } c \text{ is a constant.}$$

Given that $f(1) = 0$,

(*a*) find the value of c,

(2)

(*b*) factorise $f(x)$ completely,

(4)

(*c*) find the remainder when $f(x)$ is divided by $(2x – 3)$.

(2)

C2 June 2006

$$f(x) = 2x^3 + 3x^2 - 29x - 60.$$

(*a*) Find the remainder when f(x) is divided by (x + 2).

(2)

(*b*) Use the factor theorem to show that (x + 3) is a factor of f(x).

(2)

(*c*) Factorise f(x) completely.

(4)

C2 January 2007

$$f(x) = x^3 + 4x^2 + x - 6.$$

(*a*) Use the factor theorem to show that (x + 2) is a factor of f(x).

(2)

(*b*) Factorise f(x) completely.

(4)

(*c*) Write down all the solutions to the equation

$$x^3 + 4x^2 + x - 6 = 0.$$

(1)

C2 June 2007

$$f(x) = 3x^3 - 5x^2 - 16x + 12.$$

(*a*) Find the remainder when f(x) is divided by (x – 2).

(2)

Given that (x + 2) is a factor of f(x),

(*b*) factorise f(x) completely.

(4)

C2 January 2008

(a) Find the remainder when

$$x^3 - 2x^2 - 4x + 8$$

is divided by

(i) $x - 3$,

(ii) $x + 2$.

(3)

(*b*) Hence, or otherwise, find all the solutions to the equation

$$x^3 - 2x^2 - 4x + 8 = 0.$$

(4)

C2 June 2008

$$f(x) = 2x^3 - 3x^2 - 39x + 20$$

(*a*) Use the factor theorem to show that $(x + 4)$ is a factor of f (x).

(2)

(*b*) Factorise f (x) completely.

(4)

C2 January 2009

$$f(x) = x^4 + 5x^3 + ax + b,$$

where a and b are constants.

The remainder when f(x) is divided by $(x - 2)$ is equal to the remainder when f(x) is divided by $(x + 1)$.

(*a*) Find the value of a.

(5)

Given that $(x + 3)$ is a factor of f(x),

(*b*) find the value of b.

(3)

C2 June 2009

$$f(x) = (3x - 2)(x - k) - 8$$

where k is a constant.

(a) Write down the value of f(k).

(1)

When f(x) is divided by ($x - 2$) the remainder is 4.

(b) Find the value of k.

(2)

(c) Factorise f(x) completely.

(3)

C2 January 2010

$$f(x) = 2x^3 + ax^2 + bx - 6,$$

where a and b are constants.

When f(x) is divided by $(2x - 1)$ the remainder is -5.
When f(x) is divided by $(x + 2)$ there is no remainder.

(a) Find the value of a and the value of b.

(6)

(b) Factorise f(x) completely.

(3)

C2 June 2010

$$f(x) = 3x^3 - 5x^2 - 58x + 40.$$

(a) Find the remainder when f (x) is divided by $(x - 3)$.

(2)

Given that $(x - 5)$ is a factor of f(x) ,

(b) find all the solutions of $f(x) = 0$.

(5)

C2 January 2011

$$f(x) = x^4 + x^3 + 2x^2 + ax + b,$$

where a and b are constants.

When f(x) is divided by $(x - 1)$, the remainder is 7.

(a) Show that $a + b = 3$.

(2)

When f(x) is divided by $(x + 2)$, the remainder is -8.

(b) Find the value of a and the value of b.

(5)

C2 June 2011

$$f(x) = 2x^3 - 7x^2 - 5x + 4$$

(*a*) Find the remainder when f(x) is divided by (x – 1).

(2)

(*b*) Use the factor theorem to show that (x + 1) is a factor of f(x).

(2)

(*c*) Factorise f(x) completely.

(4)

C2 January 2012

f(x) = $x^3 + ax^2 + bx + 3$, where a and b are constants.

Given that when f (x) is divided by (x + 2) the remainder is 7,

(*a*) show that $2a - b = 6$.

(2)

Given also that when f(x) is divided by (x −1) the remainder is 4,

(*b*) find the value of a and the value of b.

(4)

C2 June 2012

$$f(x) = 2x^3 - 7x^2 - 10x + 24.$$

(*a*) Use the factor theorem to show that (x + 2) is a factor of f(x).

(2)

(*b*) Factorise f(x) completely.

(4)

C2 January 2012

$f(x) = ax^3 + bx^2 - 4x - 3$, where a and b are constants.

Given that $(x - 1)$ is a factor of $f(x)$,

(a) show that $a + b = 7$.

(2)

Given also that, when $f(x)$ is divided by $(x + 2)$, the remainder is 9,

(b) find the value of a and the value of b, showing each step in your working.

(4)

ANSWERS

C2 January 2005

(a) $a = -10$, $b = 21$

C2 June 2005

(b) $(x + 4)(2x - 1)(x - 3)$

C2 January 2006

(a) $c = 2$ (b) $(2x - 1)(x + 2)$ (c) 3.5

C2 June 2006

(a) -6 (c) $(x + 3)(2x + 5)(x - 4)$

C2 January 2007

(b) $(x + 2)(x + 3)(x - 1)$ (c) -3, -2, 1

C2 June 2007

(a) -16 (b)$(x + 2)(3x - 2)(x - 3)$

C2 January 2008

(a) (i) 5, (ii) 0 (b) $x = 2, -2$

C2 June 2008

(b) $(2x - 1)(x - 5)$

C2 January 2009

(a) $a = -20$ (b) $b = -6$

C2 June 2009

(a) -8 (b) 1 (c) $(3x - 5)(x + 2)$

C2 January 2010

(a) $a = 5$, $b = -1$ (b) $(x + 2)(2x + 3)(x - 1)$

C2 June 2010

(a) -98 (b) $\frac{2}{3}$, -4, 5

C2 January 2011

(b) $a = 9$, $b = -6$

C2 June 2011

(a) -6 (c) $(x + 1)(2x - 1)(x - 4)$

C2 January 2012

(*b*) $a = 2$, $b = -2$

C2 June 2012

(*b*) $(x + 2)(2x - 3)(x - 4)$